On the Methodological Import of Some Technical Results in Carnap's *Logical Syntax of Language*

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Two Takes on Carnap Interpretation

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While these readings are not incompatible, different interpreters tend to place emphasis upon one over the other.

My Claim:

Both readings mistake the methodological structure of Carnap's program. The technical results from *Logical Syntax* that we'll discuss today offer a case study in support of this.

Logical Empiricism

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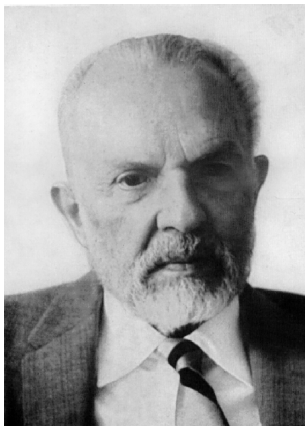
Problem:

What about logico-mathematical knowledge? Seems substantive, but not empirical—rather, it's *a priori* and necessary.

Solution:

Logico-mathematical knowledge is *not* substantive knowledge, it's Conventional: The "inferential residue" of the tacit (or explicit) syntactical rules of our language.

Conventionalist Foundations

Mathematical Conventionalism:

Mathematical languages are free of content, i.e., logico-mathematical theories are empty calculi, formal auxiliaries of our language utilized purely for the inferential manipulation of contentful, empirical sentences—thus there can be no question of absolute correctness in the choice of mathematical principles.

Rudolf Carnap
(1891–1970)

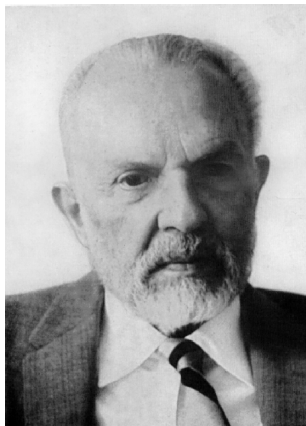# Conventionalist Foundations

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As Foundations: Logic and mathematics is nothing other than the syntax of language; logico-mathematical truths must be justified, our knowledge of them explained, and such truths generated by appeal to conventions.

Quine (1935)

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Potter (2000)

[Carnap's definition of 'Analytic' in LII requires he give up the ability to] explain how a finite intelligence can grasp arithmetical truths which appear to refer to an infinite domain of objects.

Saving Carnap's Program

The Principle of Tolerance:

It is not our business to set up prohibitions, but to arrive at conventions. [. . .]

In logic, there are no morals. Everyone is at liberty to build up his own logic, i.e., his own form of language, as he wishes. All that is required of him is that, if he wishes to discuss it, he must state his methods clearly, and give syntactical rules instead of philosophical arguments. (§17)

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The Save: Carnap's program is grounded upon Tolerance; he has retreated from traditional philosophy, instead inviting us to consider philosophical positions as a choice between frameworks, a matter of practical considerations like fruitfulness, expedience, simplicity. . .

Preliminary Remarks
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The Point

In This Talk I (Aim To) Show. . .

Carnap's conventionalism is neither a
traditional foundationalism, nor is it
grounded upon Tolerance.

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The Take Home Message:

What both interpretations miss is that Carnap provides an argument for Mathematical Conventionalism, so that the application of Tolerance to certain questions/debates is licensed by this.

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Carnap forwards Conventionalism as sufficient for the explication of our informal notion(s) of mathematical truth and of the role of logic and mathematics in the sciences.

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Overview of Logical Syntax

The Logical Syntax of Language—Overview

That part of the work of philosophers which may be held to be scientific in its nature. . . excluding the empirical questions. . . consists in logical analysis.

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The aim of logical syntax is to provide a system of concepts, a language, by the help of which the results of logical analysis will be exactly formulable. (p. xiii)

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In other words... **A Meta-Logic**

Preliminary Remarks
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Overview of Logical Syntax

Object Languages LI & LII

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LI (Safer from Contradiction) $\rightarrow$ Intuitionism

- Conservative set of axioms/inference rules
- Only bounded quantification
- Only primitive-recursive predicates/functor symbols

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Overview of Logical Syntax

Object Languages LI & LII

LI (Safer from Contradiction) → Intuitionism

- Conservative set of axioms/inference rules
- Only bounded quantification
- Only primitive-recursive predicates/functor symbols

LII (More Expressive) → Classical Mathematics

- Higher-Order Type Theory with Choice and Induction
- Unlimited quantification
- Can recover ZF set-theory.

Two Primitive Sentences of LII

Axiom of Choice (Principle of Selection)

If K is a class of classes which are mutually exclusive and non-empty, then there exists a selection class C which has exactly one element in common with every class in K.

Principle of Induction

$$\big(P(0) \bullet (x)(P(x) \supset P(x'))\big) \supset (x)(P(x))$$

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Theorems 32h.1 & 2: Choice and Induction are **Analytic** in LII.

Proof that Induction is Analytic in LII

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1. $L_{II} \models \Big(P(0) \bullet (x)\big(P(x) \supset P(x')\big)\Big) \supset (x)\big(P(x)\big)$

(Assuming FOL)

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Sahotra Sarkar (1992) concludes:

[The proofs] remain little more than formal
exercises of somewhat dubious value.

My Claim:

The proofs suggest that Carnap's goals were not the traditional foundational ones of trying to justify or generate mathematical truths; rather, his goal was the explication of mathematics and its role in empirical science.

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> Probability; Logical Truth (semantics); Theoretical Terms; Observation Sentence; Logical/Descriptive Terms; etc.

In this case, such a reading also suggests we reorient our understanding of the relationship between Tolerance and Conventionalism in Carnap's philosophical program.

What Would Carnap Say?

The proof of Theorems 1 and 2 [Induction and Choice] are interesting because they involve a fundamental question: in each one of these proofs, there is used a theorem of the syntax-language which corresponds with the theorem of the object-language whose analytic character is to be proved. (p. 121)

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So Carnap is aware of the situation, and actually finds the proofs interesting just because he's invoking meta-linguistic versions of the principles to prove them analytic.

Preliminary Remarks	A Puzzle	**WWCS?**	Explication
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Carnap's Aware of the Technical Situation

What Would Carnap Say?

It is clear that the possibility of proving a certain syntactical sentence depends upon the richness of the syntax-language which is used, and especially upon what is regarded as valid in this language. In the present case, the situation is as follows: we can work out in our syntax-language S (for which we have here taken a not strictly determined word-language) the proof that a certain sentence, $\mathfrak{S}_1$, of the object-language II is analytic, if, in S, we have a certain sentence at our disposal, namely, that particular sentence of S which (in ordinary translation) is translatable into the sentence $\mathfrak{S}_1$ of II. From this it follows that our proof is not in any way a circular one. (pp. 123–124)

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But this still doesn't make it a **useful** one.

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Carnap's Explanation

The proofs of Theorems 1 and 2 [Induction and Choice] must not be interpreted as though by means of them it were proved that the Principle of Induction and the Principle of Selection were materially true. They only show that our definition of 'analytic' effects on this point what it is intended to effect, namely, the characterization of a sentence as analytic if, in material interpretation, it is regarded as logically valid. (p. 124)

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> The correct extension of 'classical logico-mathematical truth' in an unreconstructed or informal sense.

The question as to whether the Principle of Selection should be admitted into the whole of the language of science (including also all syntactical investigations) as logically valid is not decided thereby. That is a matter of choice, as are all questions concerning the language-form which is to be chosen (cf. the Principle of Tolerance, §17 and §78). In view of the present knowledge of the syntactical nature of the Principle of Selection, its admission should be regarded as expedient. (p. 124)

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We are justified in our Tolerant attitude toward Choice because it is considered logically valid in an informal or unreconstructed sense.
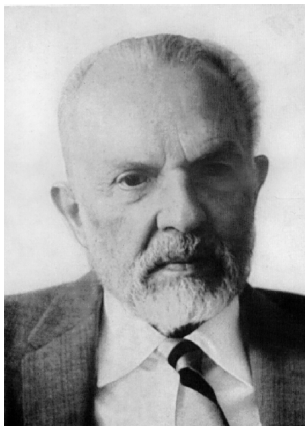
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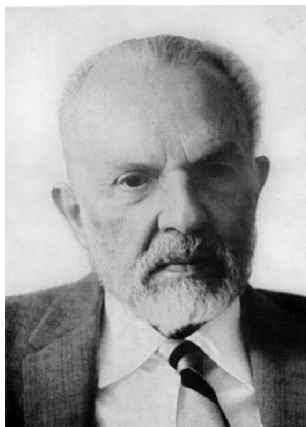
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We are justified in our Tolerant attitude toward Choice because it is considered logically valid in an informal or unreconstructed sense.

So Carnap's assuming the validity of Ax. Choice—this is reason enough to treat it syntactically (as a formal auxiliary)—our ability to treat it as such then allows us free choice to include or exclude it in our (formally reconstructed) language.

Preliminary Remarks
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Carnap's Explanation

The Lesson

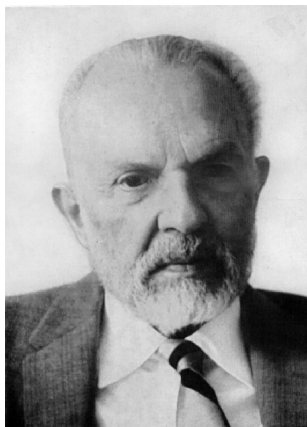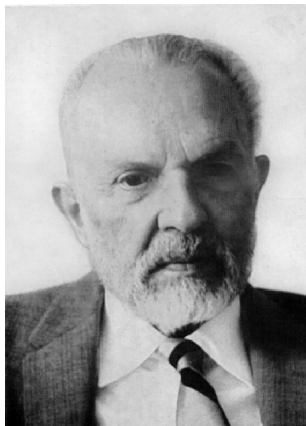As an absolute or foundational justification of the principles, the proofs are useless.

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Notice: Carnap takes the validity classical mathematics as a given, and invokes the Principle of Tolerance only on the basis of this—his reconstruction is constrained by our informal notions.

In other words, he is offering an **Explication**.

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Carnap's Program isn't Grounded on Tolerance

Carnap: A *Scientific* Philosophy

The Task of the Logic of Science (1934)

[O]ur own discipline, logic or the logic of science, is in the process of cutting itself loose from philosophy and of becoming a properly scientific field, where all work is done according to strict scientific methods and not by means of "higher" or "deeper" insights.
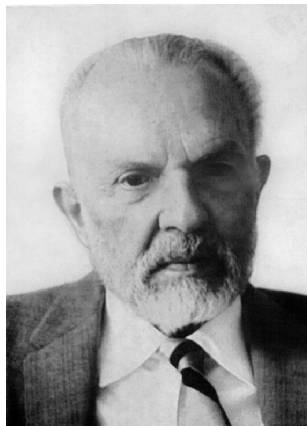
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Carnap wants to replace traditional philosophical methods with the logical analysis of the concepts, methods, and languages of science.

Not Tolerance all the way down: Must take the methods and results of science seriously.

The Explication of the Concepts of Science
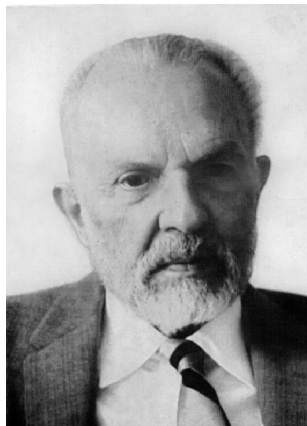

1961 Preface to the *Aufbau* (1928)

By rational reconstruction is here meant the searching out of new definitions for old concepts. The old concepts did not ordinarily originate by way of deliberate formulation, but in more or less unreflected and spontaneous development. The new definitions should be superior to the old in clarity and exactness, and, above all, should fit into a systematic structure of concepts. Such a clarification of concepts, nowadays frequently called "explication", still seems to me one of the most important tasks of philosophy, especially if it is concerned with the main categories of human thought.

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Explaining Explication

The Explication of the Concepts of Science

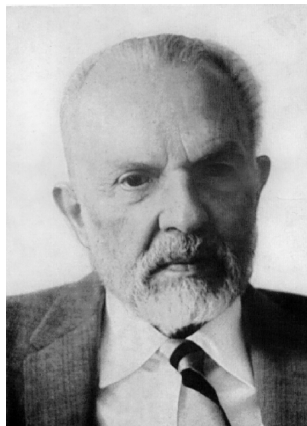Carnap most frequently sites:

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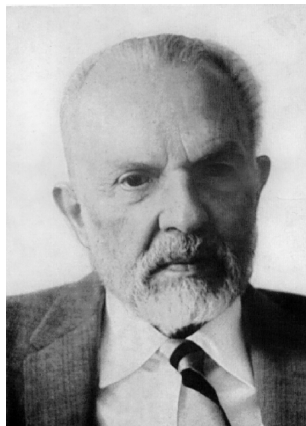- The Frege-Russell definition of 'Number' and the numerals
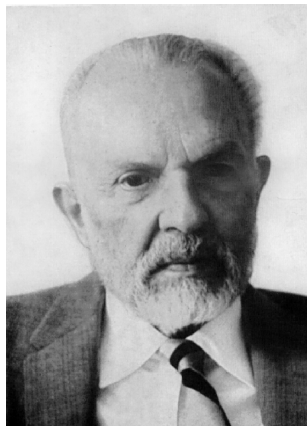
The Explication of the Concepts of Science

Carnap most frequently sites:

- The Frege-Russell definition of 'Number' and the numerals
- Russell's analysis of definite descriptions

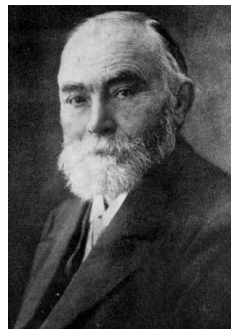# The Explication of the Concepts of Science

Carnap most frequently sites:

- The Frege-Russell definition of 'Number' and the numerals
- Russell's analysis of definite descriptions
- Tarski's definition of 'Truth' for formal languages

Preliminary Remarks
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Explaining Explication

The Explication of the Concepts of Science

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- Russell's analysis of definite descriptions
- Tarski's definition of 'Truth' for formal languages

What's common to all of these examples is that the authors attempt to show their explications adequate by arguing that they recover essential characteristics of the informal concepts.

Obligatory Frege Quotation

Grundlagen der Arithmetik, §**70 (1884)**

Definitions show their worth by proving fruitful. [. . .] Let us try, therefore, whether we can derive from our definition of *the Number which belongs to the concept F* any of the well-known properties of numbers.
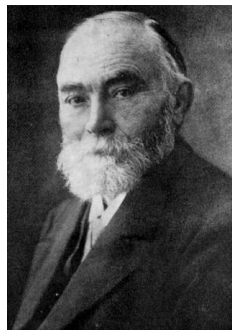

Gottlob Frege
(1848–1925)

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Frege suggests: **Generality**, **Objectivity**, and the
typical **Theorems/Properties**.
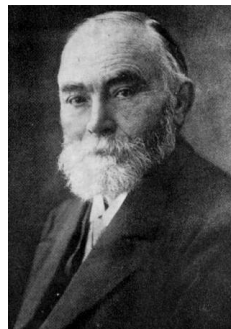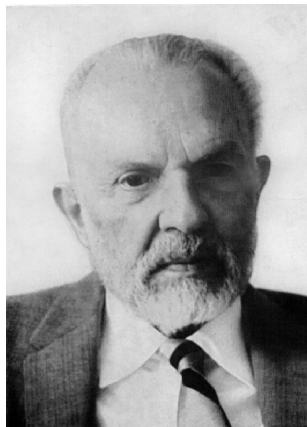

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Frege suggests: **Generality**, **Objectivity**, and the
typical **Theorems/Properties**.

Carnap's doing something similar: He proposes LII
as a reconstruction of a mathematical theory, and
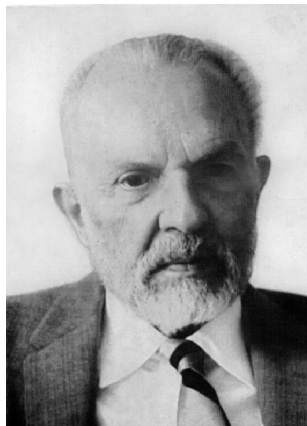then argues that it is adequate in that regard.

Gottlob Frege
(1848–1925)

Preliminary Remarks
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A Methodological Argument for Conventionalism

Conventionalism as an Explication

I want to suggest that Carnap is also forwarding Mathematical Conventionalism itself in this same light:

Conventionalism as an Explication

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Mathematical Conventionalism itself in this
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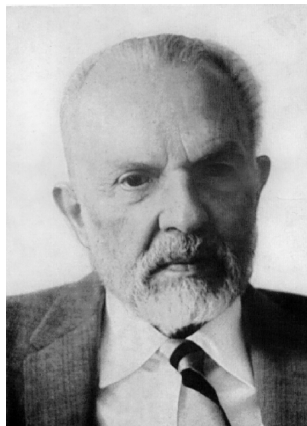> characteristics of mathematical
> practice.

Conventionalism as an Explication

I want to suggest that Carnap is also forwarding Mathematical Conventionalism itself in this same light:

> As an explication of role of mathematics in the sciences, or as adequate to recover the essential characteristics of mathematical practice.

Carnap Suggests: **Applicability**, and mathematics' **Role in the Sciences** (i.e., methods, prediction/explanation)

Tolerance in Mathematics

Carnap argues that the Principle of Tolerance is a typical mathematical attitude:

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mathematical attitude:

Logical Syntax, §17

The tolerant attitude here suggested is, as far as special
mathematical calculi are concerned, the attitude which is tacitly
shared by the majority of mathematicians.

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A Methodological Argument for Conventionalism

Tolerance in Mathematics

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mathematical attitude:

Logical Syntax, §**17**

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mathematical calculi are concerned, the attitude which is tacitly
shared by the majority of mathematicians.

The idea is that most mathematicians are happy to explore the
consequences of various mathematical theories or the properties of
various mathematical structures, using whatever methods of proof
so long as they state their assumptions clearly.

The Role of Mathematical Sentences

Carnap Asks: What should a logical foundation
of mathematics achieve?

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of mathematics achieve?

Foundations of Logic and Mathematics

The chief function of a logical calculus in its
application to science is not to furnish logical
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deduction of factual conclusions from factual
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The Role of Mathematical Sentences

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> ### Foundations of Logic and Mathematics
>
> The chief function of a logical calculus in its application to science is not to furnish logical theorems, i.e., L-true sentences, but to guide the deduction of factual conclusions from factual premises. (p. 35)

Carnap argues that a logical interpretation of mathematical sentences achieves the requisite generality, and that within the context of a language including also descriptive symbols and synthetic sentences it accounts for this role.

Tolerance and Conventionalism

Again, Carnap is assuming that classical mathematics is valid, so far as it goes, and trying to show that a Conventionalist account is adequate to capture its role in the sciences.

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A Methodological Argument for Conventionalism

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This is what it means to say that Carnap takes the results of the sciences seriously: *Philosophers have no special authority to tell mathematicians which methods or principles are correct.*

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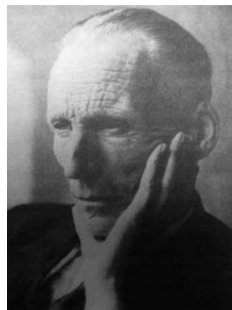
This is what it means to say that Carnap takes the results of the sciences seriously: *Philosophers have no special authority to tell mathematicians which methods or principles are correct.*

But we can still contribute via explication, to make concepts clearer or help to understand their essential characteristics.

Preliminary Remarks
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A Methodological Argument for Conventionalism

Tolerance and Conventionalism

Why a Conventionalist Account?

Because it allows us to invoke Tolerance in our choice of logico-mathematical principles.

L.E.J. Brouwer
(1881–1966)

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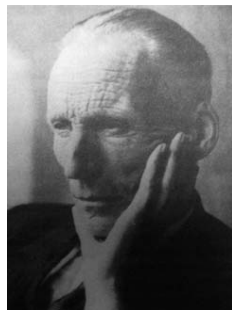
If conventionalism is sufficient to account for mathematics in science, we can transform questions of absolute validity into questions of the inferential methods we prefer.
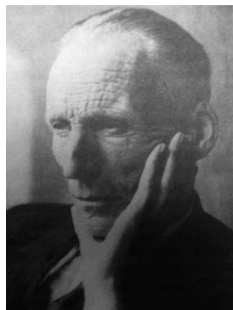

L.E.J. Brouwer
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Tolerance and Conventionalism

Why a Conventionalist Account?

Because it allows us to invoke Tolerance in our choice of logico-mathematical principles.

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So we get an advantage of method over competing accounts (everything else is non-science).

L.E.J. Brouwer
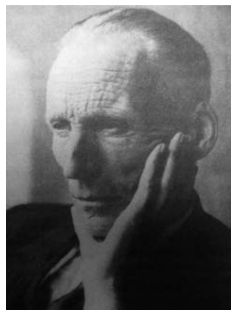(1881–1966)

Tolerance and Conventionalism

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If conventionalism is sufficient to account for mathematics in science, we can transform questions of absolute validity into questions of the inferential methods we prefer.

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To the Intuitionist: We don't defer to Tolerance until after we've made plausible that conventionalism is indeed sufficient.

L.E.J. Brouwer
(1881–1966)

Preliminary Remarks
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Conclusions

Summing Up

Scientific Philosophy: The Explication of the concepts, methods, theories, and languages of the sciences, including mathematics.

Preliminary Remarks
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Summing Up

Scientific Philosophy: The Explication of the concepts, methods, theories, and languages of the sciences, including mathematics.

Logical Syntax tries to develop a formal and systematic context for performing explications.

Summing Up

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Summing Up

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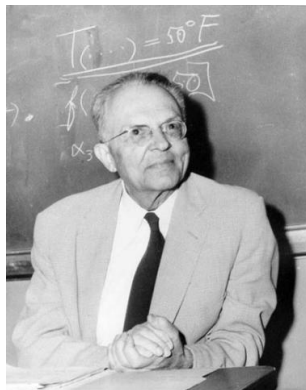Carnap supports his Conventionalism with separate, methodological arguments

Summing Up

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Logical Syntax tries to develop a formal and systematic context for performing explications.

The proofs act as evidence for LII as an adequate explication of classical mathematical truth.

Carnap supports his Conventionalism with separate, methodological arguments—is itself an explication of the role of mathematics in the sciences.

Preliminary Remarks
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Thanks!