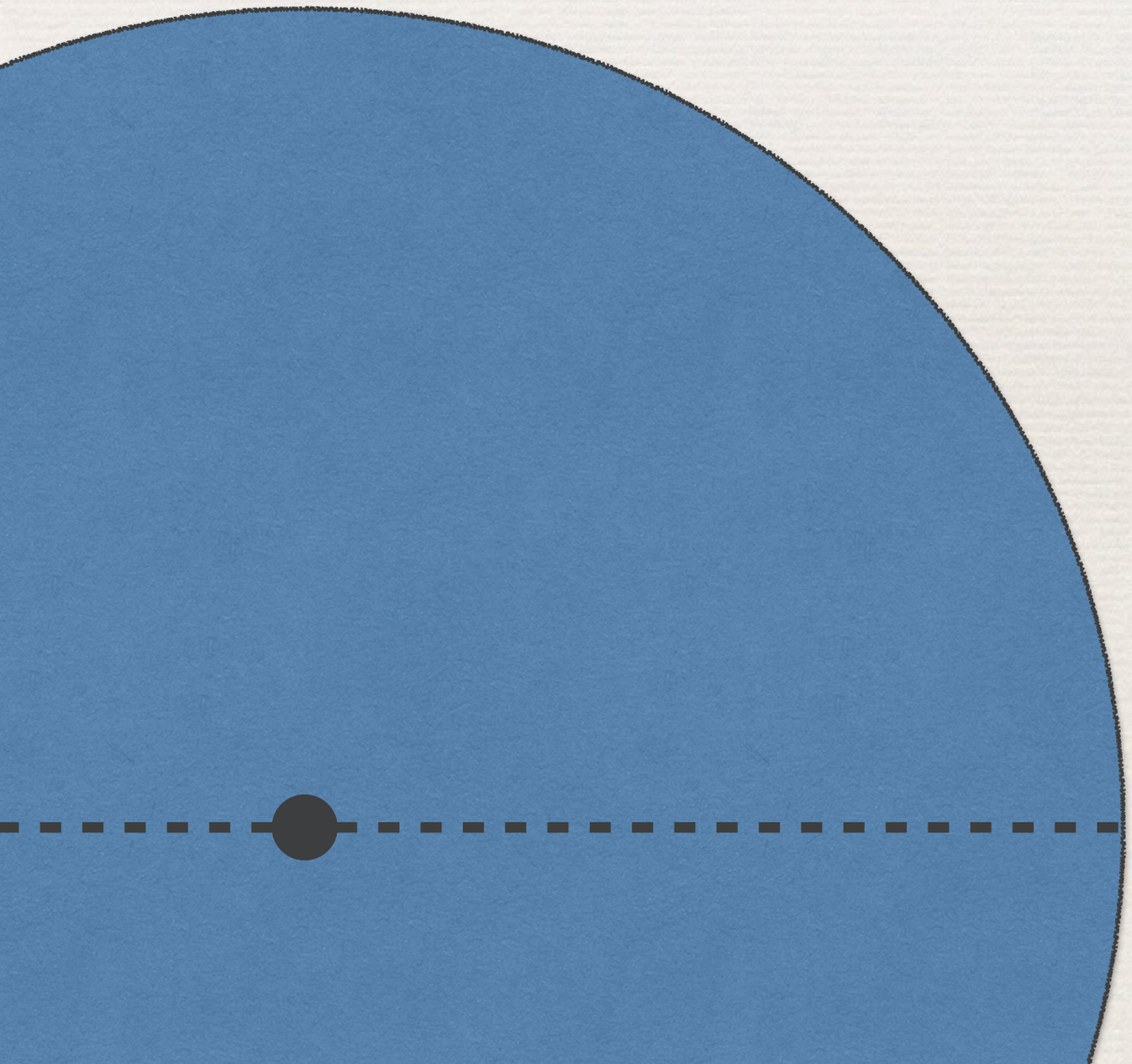
Ancient and Medieval Astronomical Theory & Method

Emerson Doyle, January 15, 2016
for the Royal Astronomical Society of Canada - London Centre

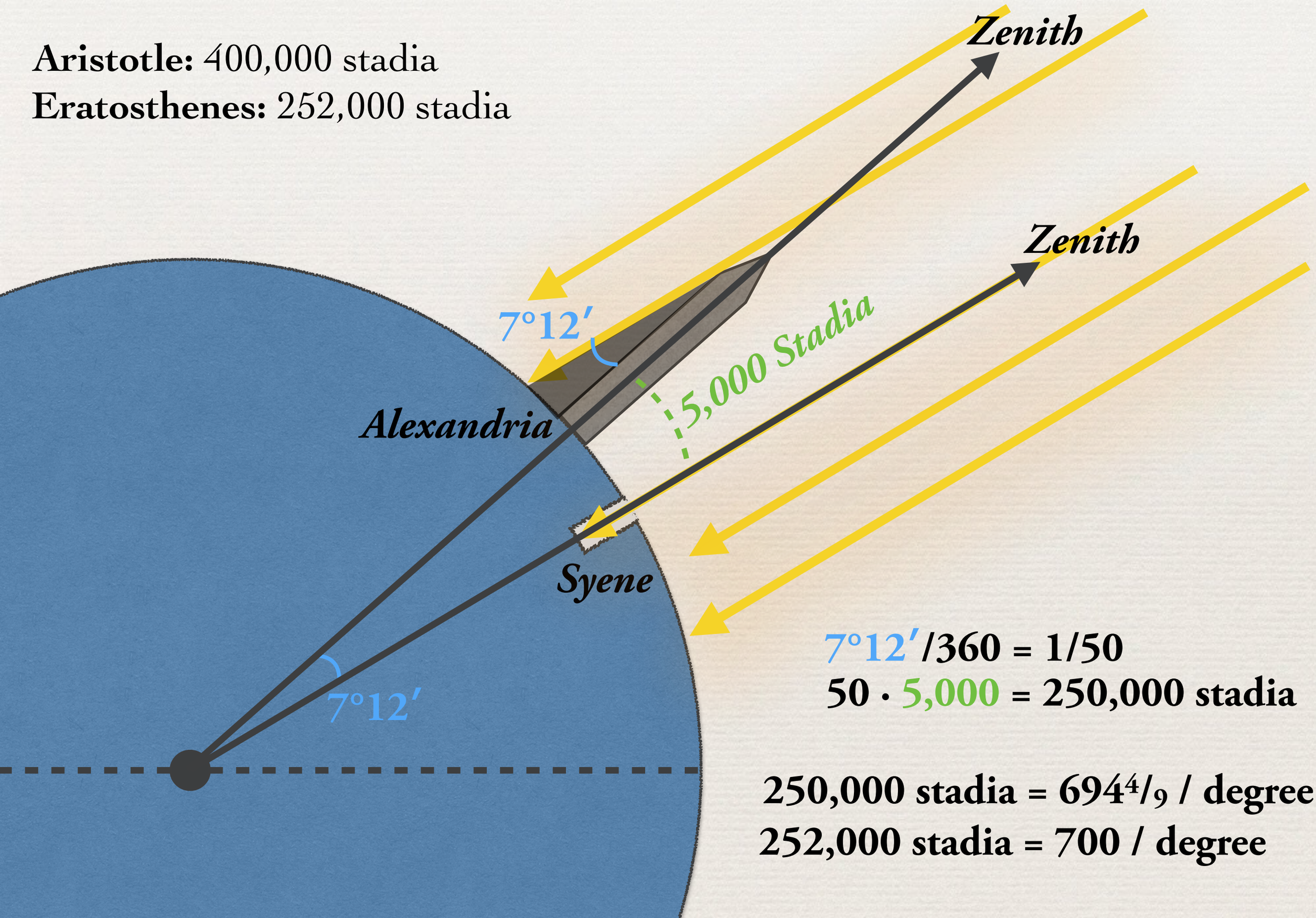
The Earth

Aristotle: 400,000 stadia



1. During a lunar eclipse, the Earth casts a convex shadow.
2. Stars visible from Egypt are not visible at more northern latitudes.
3. Ships at sea seem to "dip below" the horizon on calm days—the same in all directions.
4. "Heavy" bodies naturally move toward the centre of the universe.

Aristotle: 400,000 stadia
Eratosthenes: 252,000 stadia



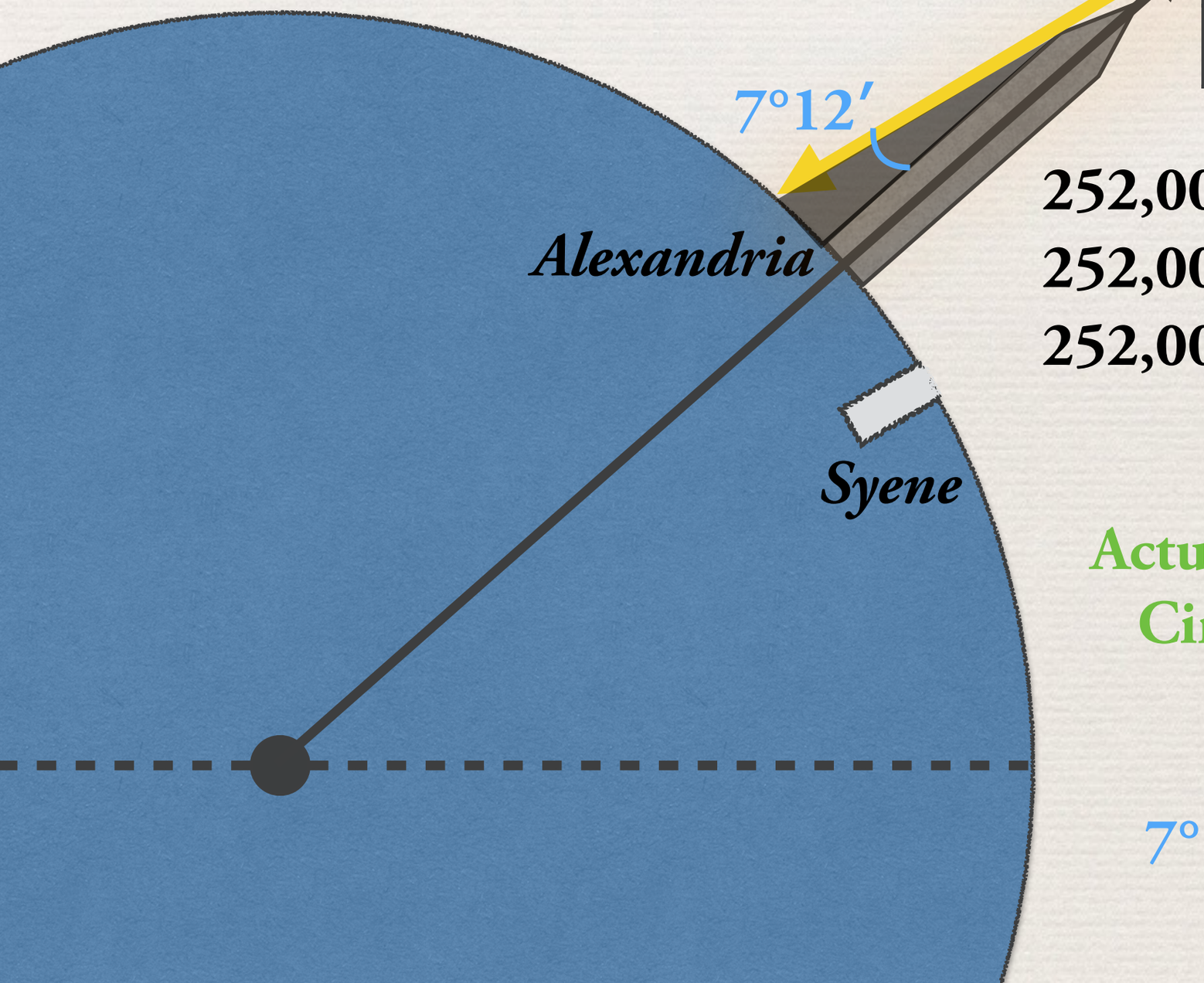
$7°12′/360 = 1/50$
$50 \cdot 5{,}000 = 250{,}000$ stadia

250,000 stadia = $694\,4/9$ / degree
252,000 stadia = 700 / degree

How Accurate?

Eratosthenes: 252,000 stadia

Itinerary	157 metres
Babylonian	196 metres
Ptolemaic	185 metres



252,000 · 157 m = 39,564 km (1.2%)
252,000 · 196 m = 49,392 km (19%)
252,000 · 185 m = 46,620 km (14%)

Actual Meridional Circumference = 40,007 km

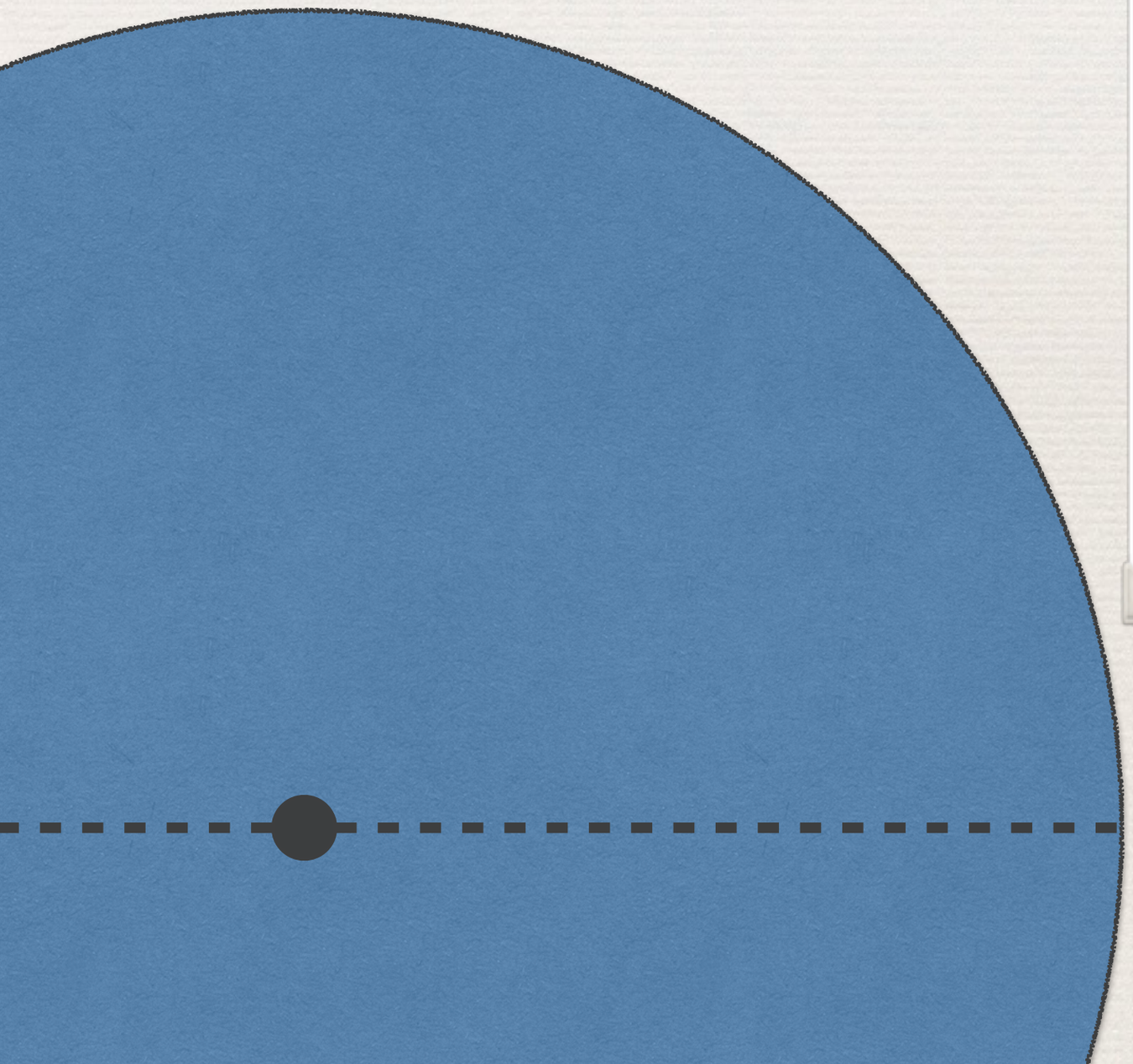
7°12′ about 16′ too small (4%)

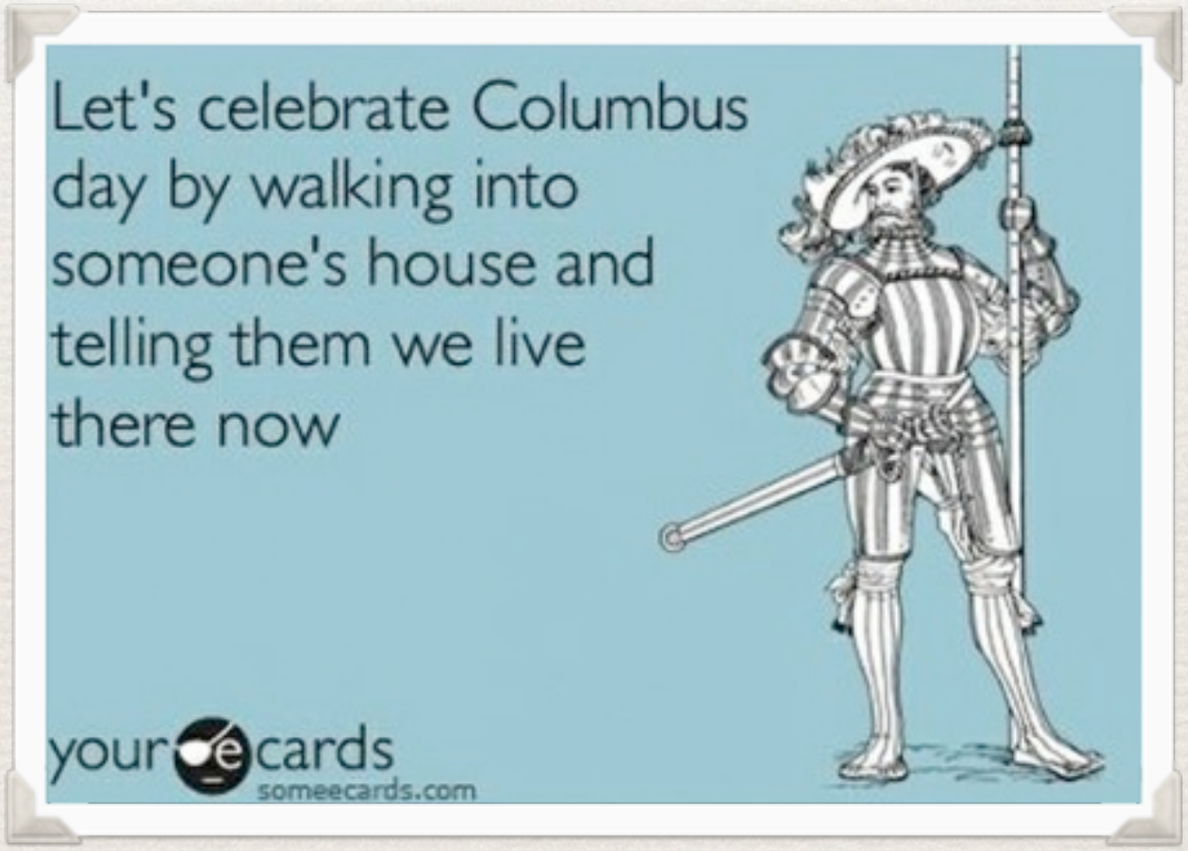
How Accurate?

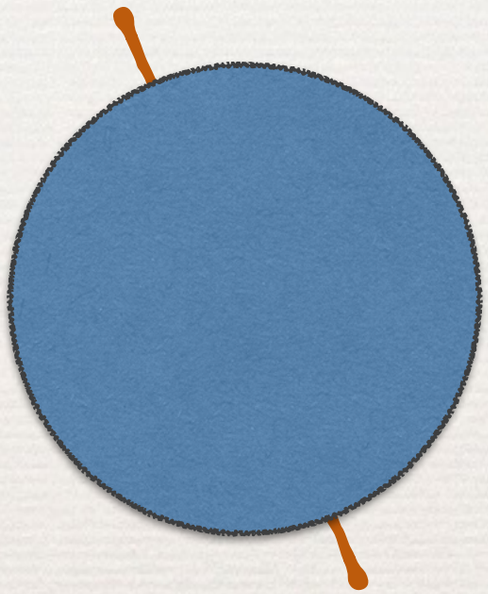
7°12′ about 16′ too small (4%)

How Accurate?

Eratosthenes: 252,000 stadia
Posidonius (51 BCE): 180,000 stadia





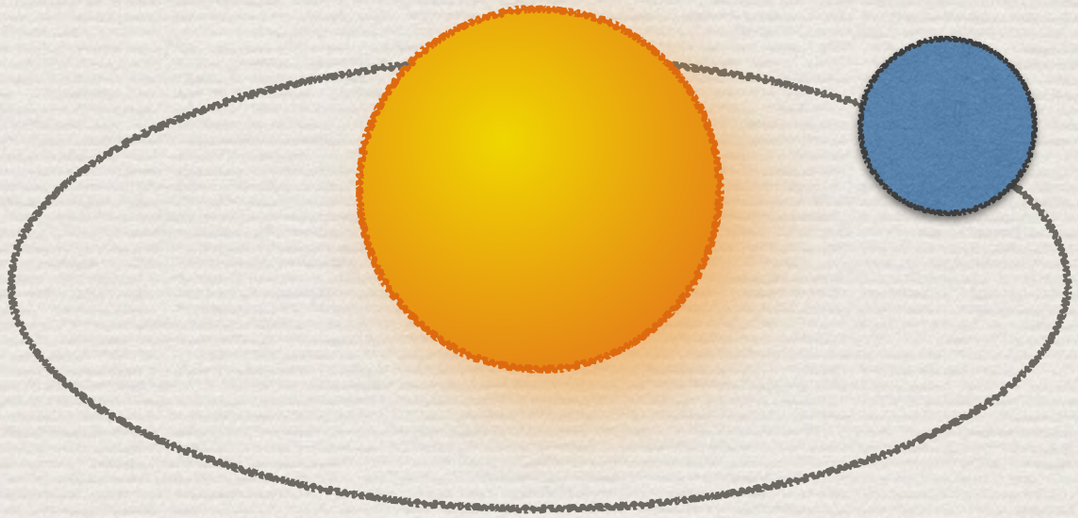


- Idea rejected due to a lack of "rotational wind".
- And disagreement with the accepted physics of the day.
  - Aristotelian physics supposes that a bird in the air or object dropped/thrown would be "left behind" to the west as the Earth moved at great speed.

Heraclides Ponticus
(390 - 310)

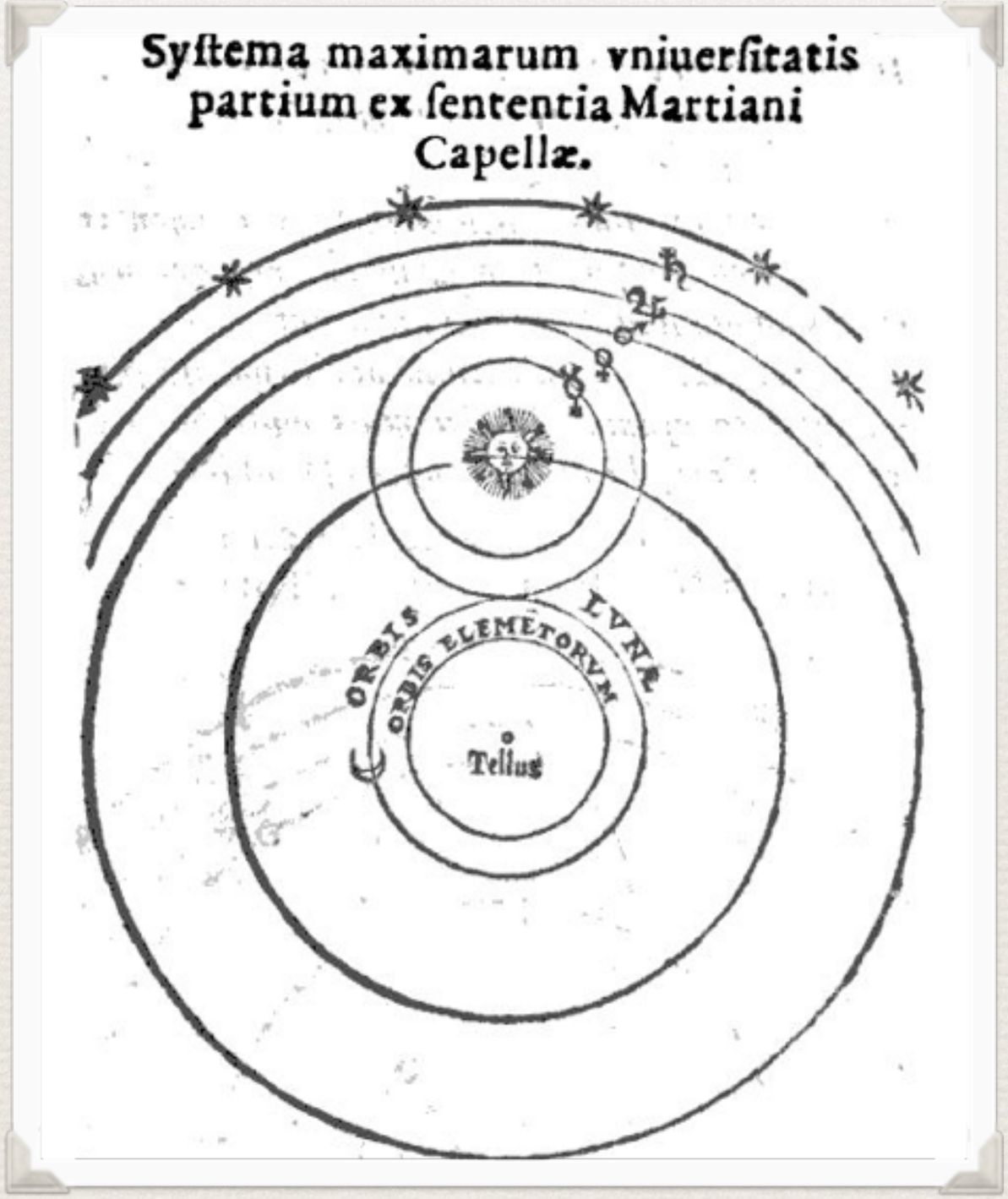
- Entirely possible that this theory was considered for several hundred years.
  - Lack of stellar parallax discussed.
- Again, disagrees with accepted Aristotelian physics of the day.

Aristarchus of Samos
(310 - 230 BCE)



Martanius Capella's *On the Marriage of Philology and Mercury,* or *On the Seven Disciplines*

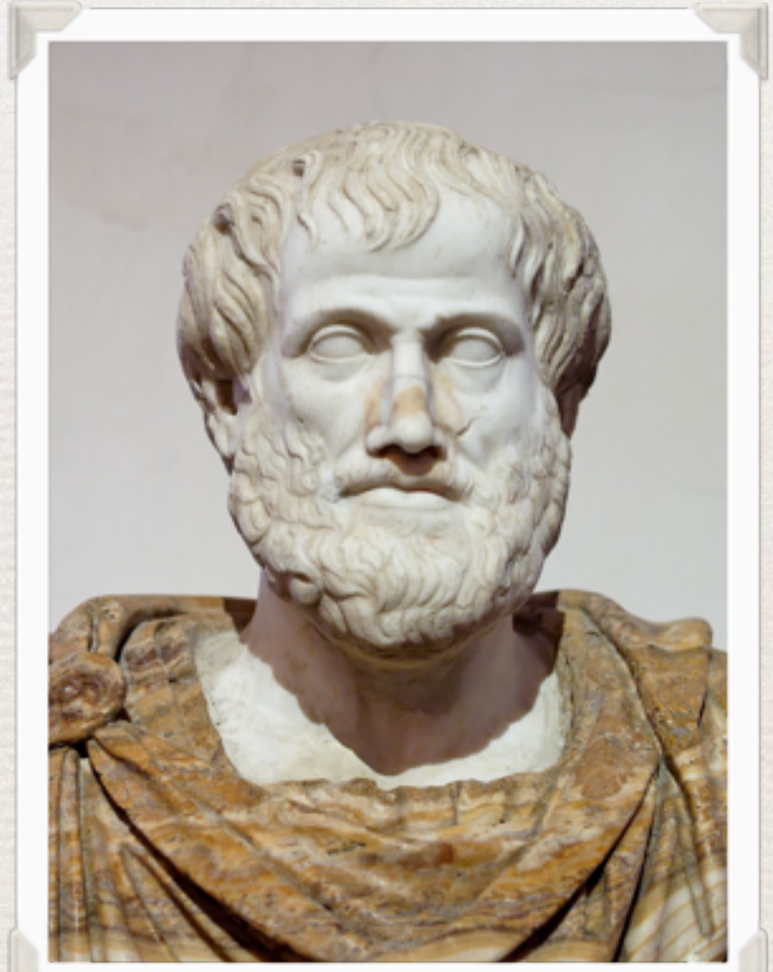
- Helps explain why Mercury & Venus are observed to "travel with the Sun".
  - (I.e., Mercury & Venus' greatest elongations are between 18° and 28°, and between 45° and 47° respectively)



(Naboth's 1573 representation of Capella's world system)

Aristotelian Physics

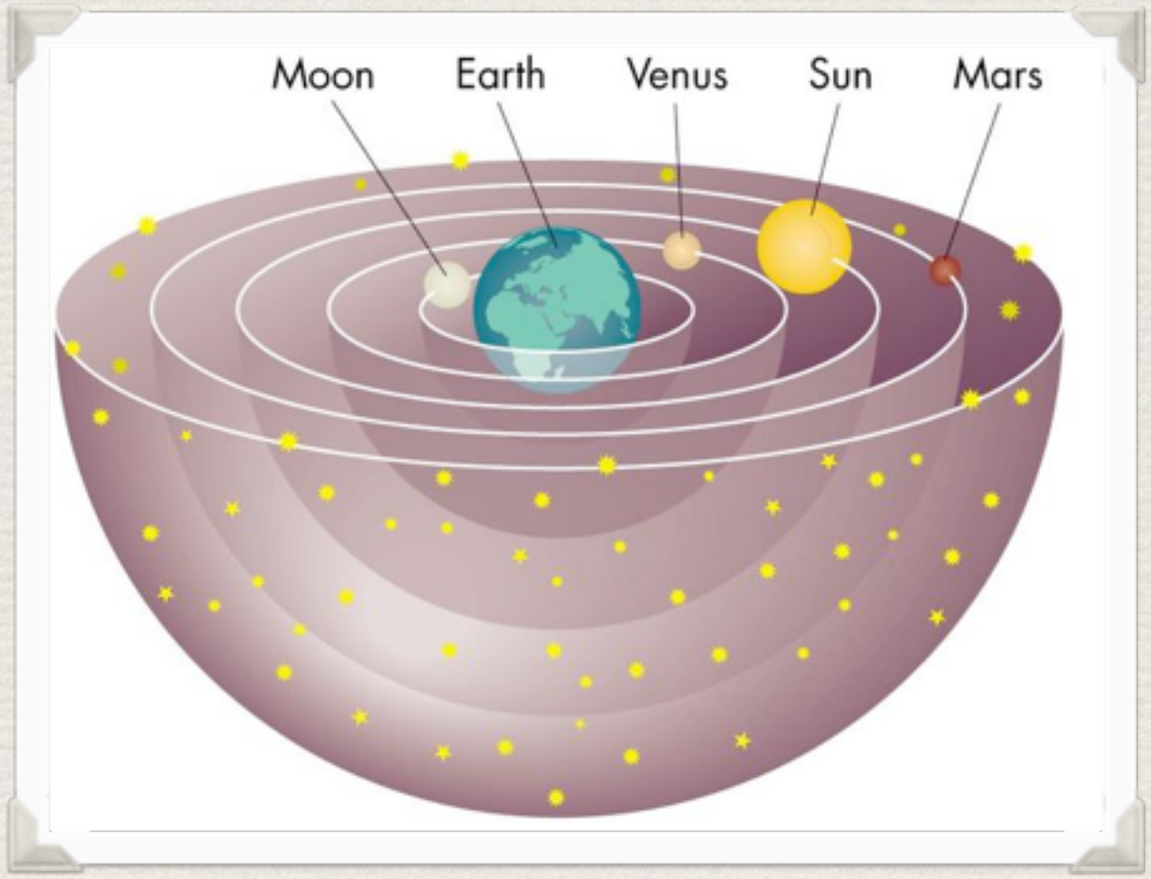
- Aristotelian physics is **Teleological** rather than Mechanical—things have a **purpose**.
  - The purpose of objects in the celestial sphere is to mark the passage of time, or even to generate it by their motion.
- Distinction between motion **Forced** & **Natural**.
  - Circular/Spherical motion is the most perfect kind of motion, and so the only natural type suitable to celestial objects.
- The universe is **Rational** and **Knowable**—some sort of assumption along these lines is a **Precondition** for doing science.

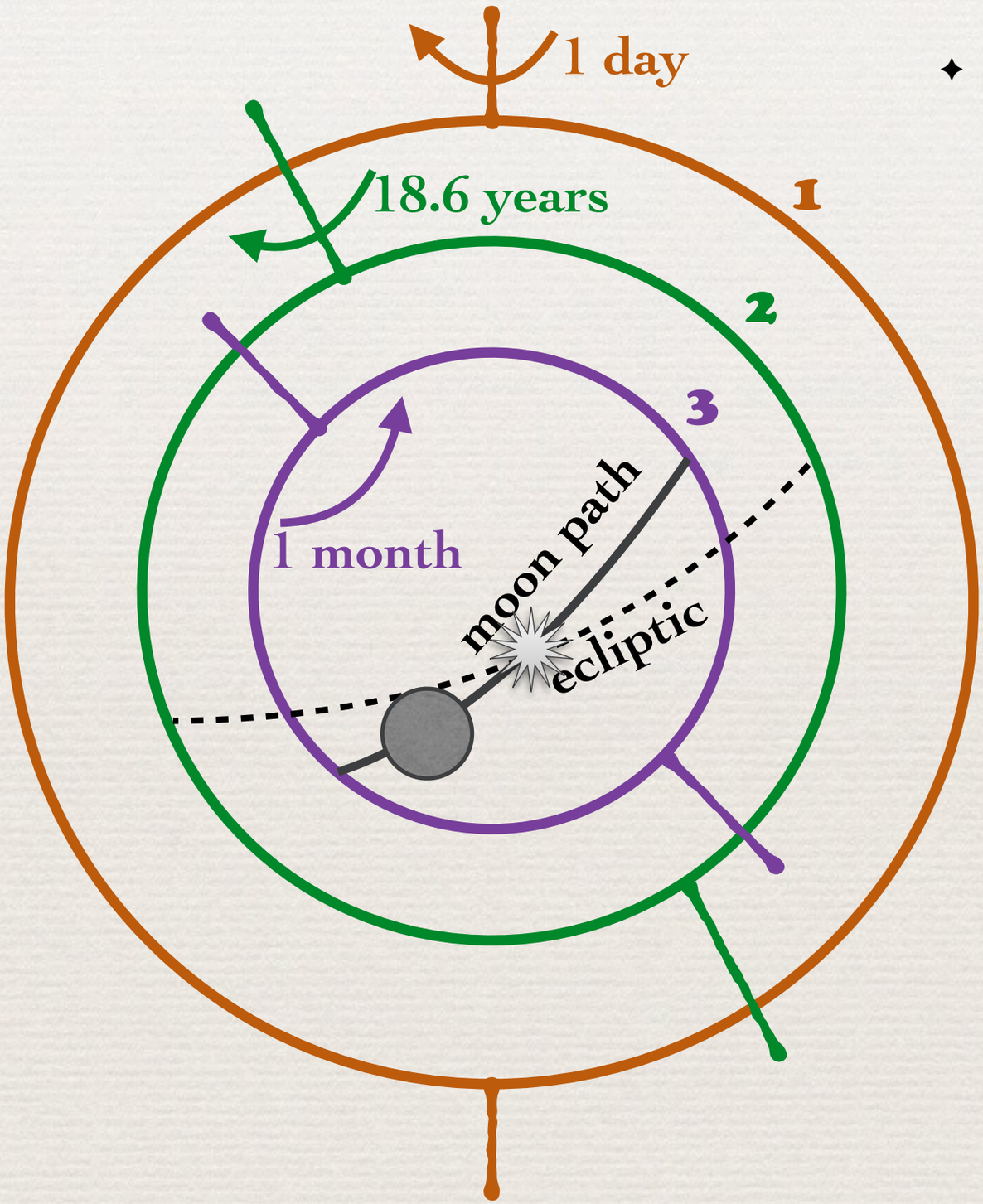


Aristotle
(384 - 322 BCE)

Aristotelian Cosmology

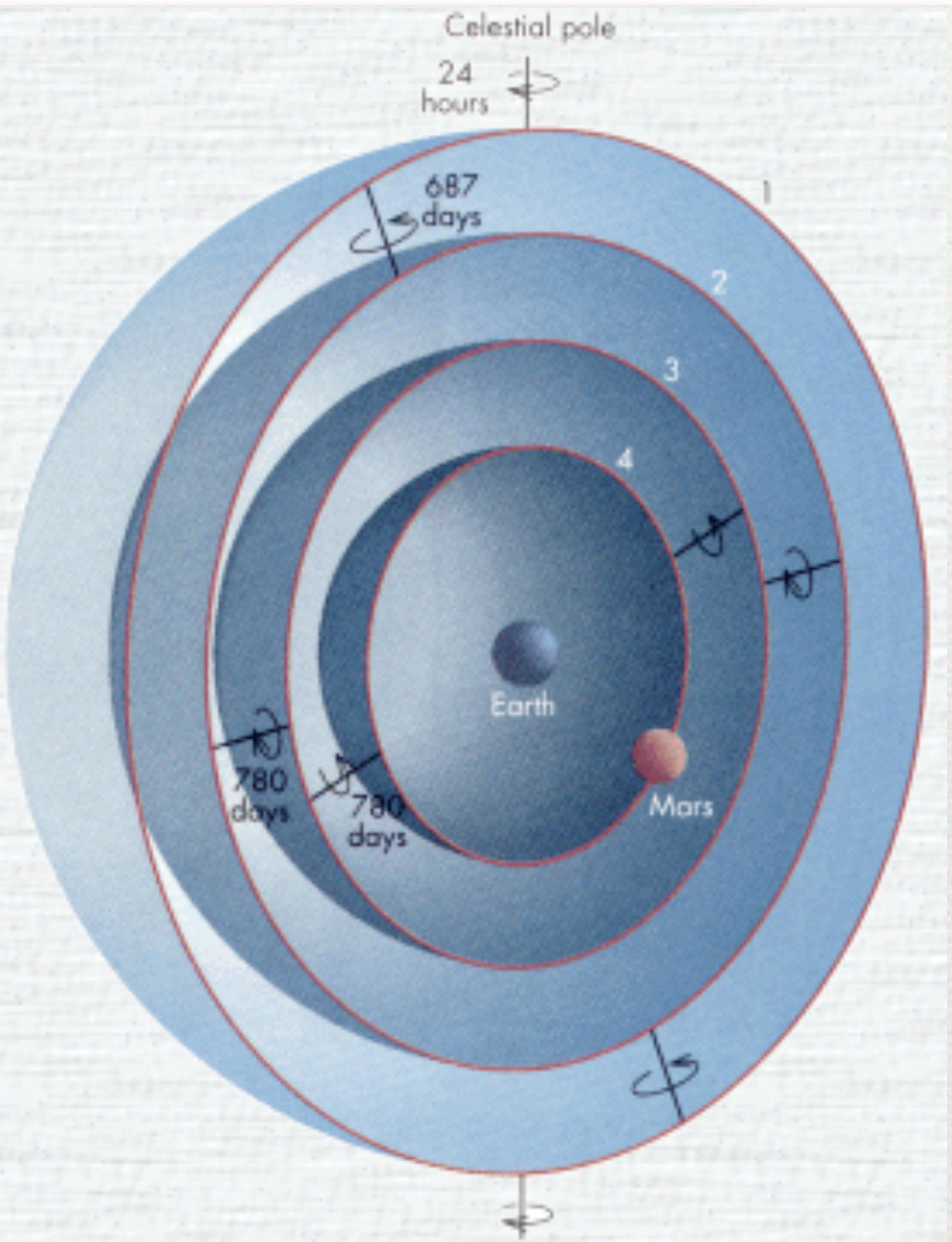
- **The Assumption:** In other words, the cosmos should work according to relatively simple principles.
    - We should be able to recover the complex and apparently random phenomena we observe via natural, universal, and consistent principles.
- **The Methodology:** The apparently random, non-uniform motions of the celestial realm should be **reducible** to some compound set of uniform circular motions.
    - Drives astronomical work for the next 1800 years.





Theory of the Moon

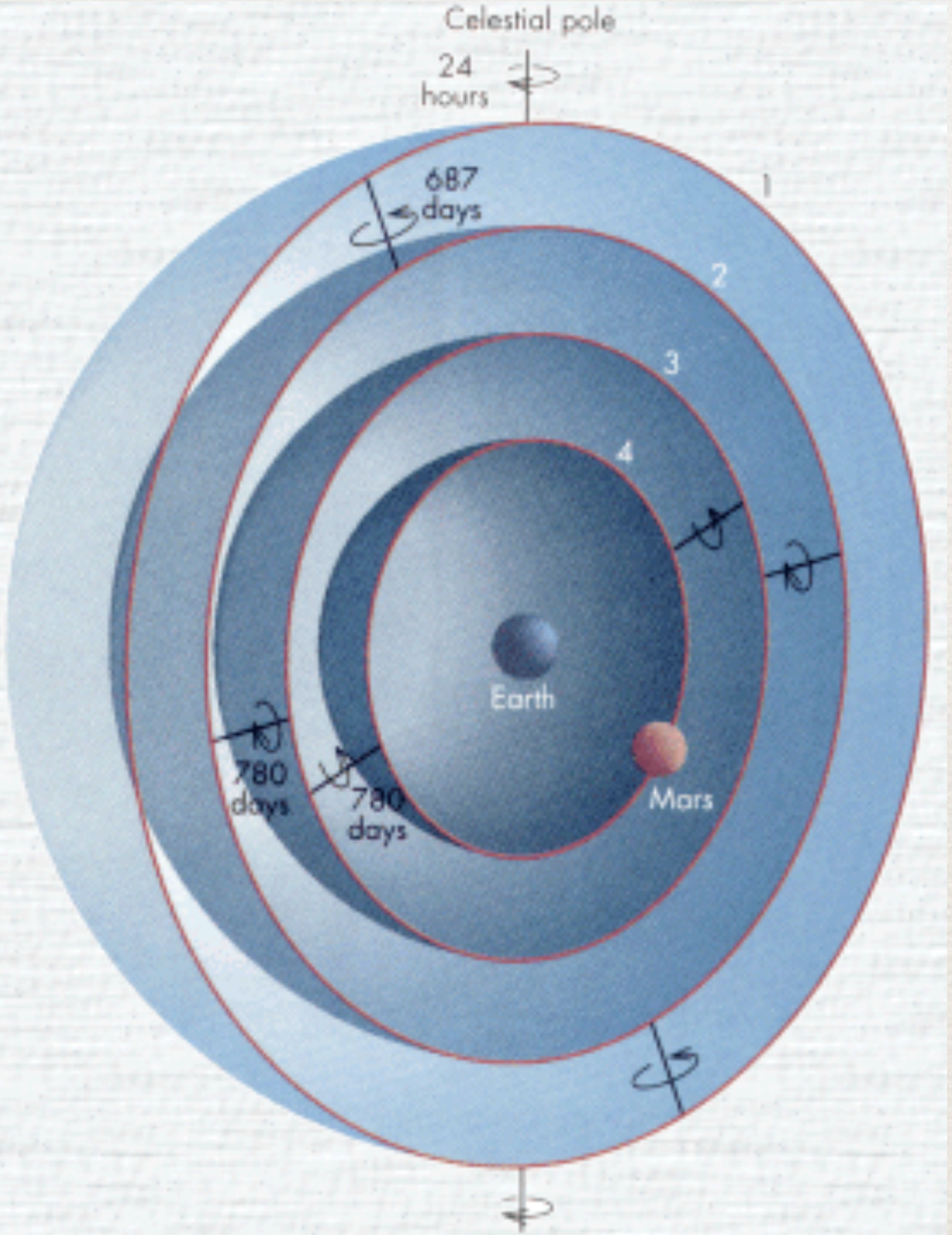
- **Eudoxus of Cnidus** (408 - 355 BCE) proposes a "nested-sphere" model.
  - Sphere 2 accounts for the fact that successive eclipses do not occur at the same spot—the nodes of the Moon's orbit work their way westward every 18.6 years.
- **Explains:** Daily motion of the moon; change in longitude around the ecliptic; motion in latitude; and the displacement of successive eclipses around the zodiac.
- Eudoxus also uses 3 spheres for the Sun, indicating that they were unclear about the nature of the ecliptic (likely due to observational error).



Eudoxus' model for the motion of Mars. The inner two spheres produce retrograde motion, sphere 2 moves the planet eastward, and sphere 1 accounts for the daily rising and setting of Mars.

- For the planets, we must also account for retrograde motion.
- Spheres 3 and 4 create a **hippopede** (figure-eight path) as a result of two opposite offset motions.
- The system is rough, offering some **qualitative** accuracy—unlikely he assigned an angle between 3 and 4.

Demonstration



Eudoxus' model for the motion of Mars. The inner two spheres produce retrograde motion, sphere 2 moves the planet eastward, and sphere 1 accounts for the daily rising and setting of Mars.

- **Some problems:**
  - Doesn't account for changes in brightness; planets cross ecliptic four times/synodic period, not twice.
  - Model should be seen as a physical allegory, consistent with Greek cosmological principles; and a forum for proving geometric theorems.
  - Doesn't account for the Sun and Moon anomalies.
- Aristotle argues that the model is a **real working mechanism**—introduces "counteracting" spheres.

Babylonian Observational Astronomy

- Systematic observations date back to at least 1800 BCE—E.g., dates of the new moon, a rather complete set of risings/settings of Venus over 21 years.
- Starting around 700 BCE and until about 50 BCE, we find rather complete sets of clay tablet "Almanacs" recording sophisticated observations of the stars and planets for the preceding six (or seven) month period:
  - Number of days in the preceding months; the time between moonrise and sunset or sunset and moonrise dependent upon the date
  - Longitudes of each of the planets (the signs of the zodiac in which they are visible); date of first/last visibility; dates of retrogradation; opposition
  - Details of eclipses; conjunctions of moon/planets with stars near zodiac
  - The river level in Babylon; the price of barley, dates, sesame, etc.; the weather; and interesting news.

- The Babylonian theory of celestial motions was purely arithmetic—they made no attempt to develop geometric models or perform any kind of philosophical or physical speculation to explain motions.
  - Instead they were interested in **predictive accuracy**. This was originally accomplished with "*Goal-Year Texts*" on the basis of the Almanacs:

Venus

Tropical Period	**Synodic Period**
Eastward around the zodiac	Responsible for retrogradation
365 1/4 days · 8 = 2,922 days	**584 days · 5 = 2,920 days**

- A very good cycle for Venus is thus **8 years**—if we want to predict the behaviour of Venus in a given year (our "Goal Year"), we look up its behaviour 8 years previous.

Great Cycles

	Cycle Length	Tropical Period	Synodic Period
Jupiter (phenomena)	71 years	= 6 tropical	= 65 synodic
Jupiter (passages)	83 years	= 7 tropical	= 76 synodic
Venus	8 years	= 8 tropical	= 5 synodic
Mercury	46 years	= 46 tropical	= 145 synodic
Saturn	59 years	= 2 tropical	= 57 synodic
Mars (passages)	47 years	= 25 tropical	= 22 synodic
Mars (phenomena)	79 years	= 42 tropical	= 37 synodic
Moon	18 years		

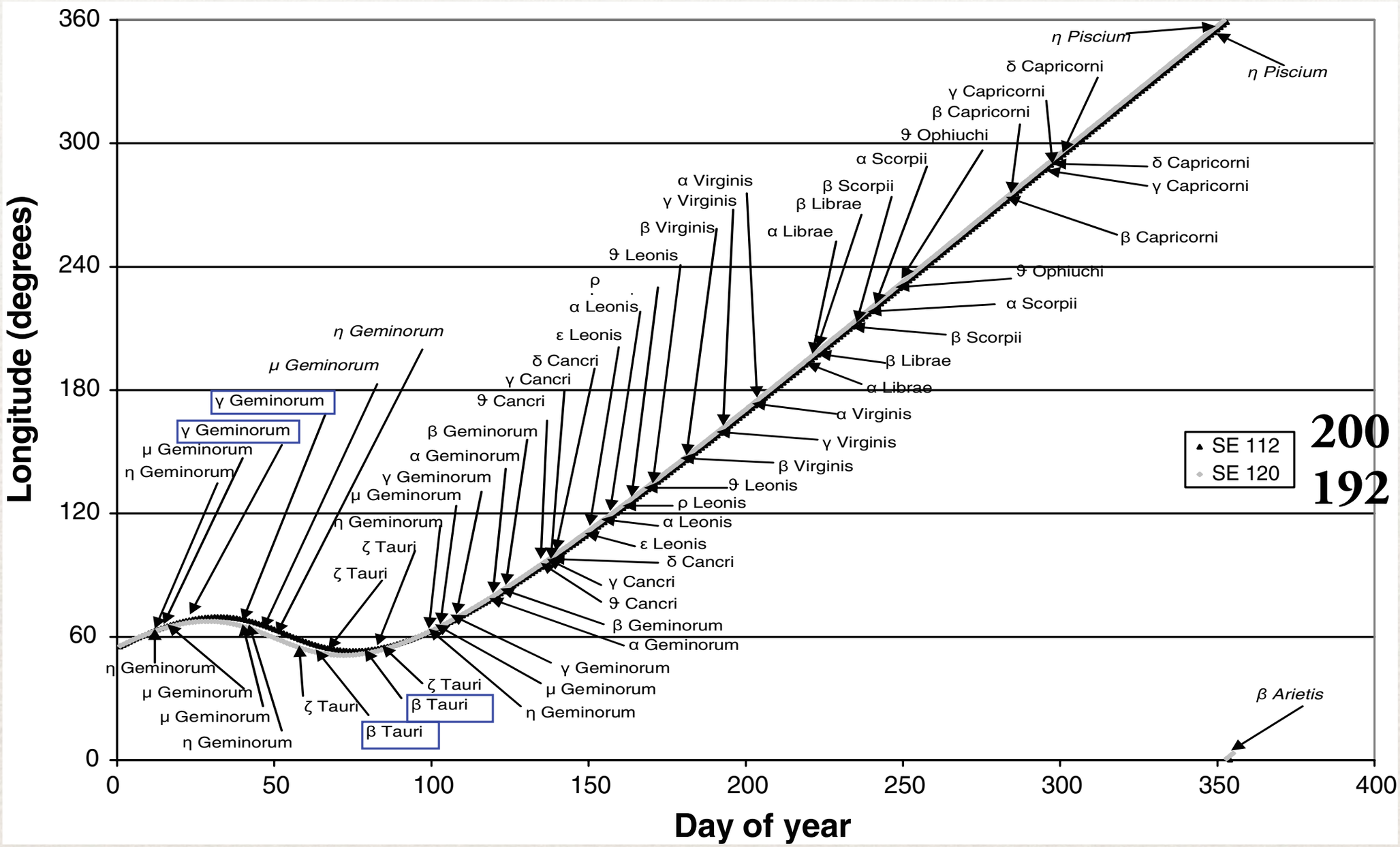


Fig. 4 Venus' longitude during the Babylonian years SE 112 and SE 120, marking the points where the planet passed by each Normal Star. Star names in italics were passed by during Venus' periods of invisibility. The boxes around γ Geminorum and β Tauri highlight stars which the planet passed by in one year but not the other, showing that these planetary passages could not have been predicted using Goal-Year Text methods

J. M. K. Gray, J. M. Steele (2008). "Studies on Babylonian goal-year astronomy I: A comparison between planetary data in Goal-Year Texts, Almanacs and Normal Star Almanacs", *Arch. Hist. Exact Sci.*

- This notion of cycles was also applied to the calendar, giving a luni-solar calendar of 19-year cycles, called a **Metonic Cycle** in Athens:

12 years of 12 months (29/30 days) = 144 months

7 years of 13 months (29/30 days) = 91 months

Thus 19 calendar years = 235 months

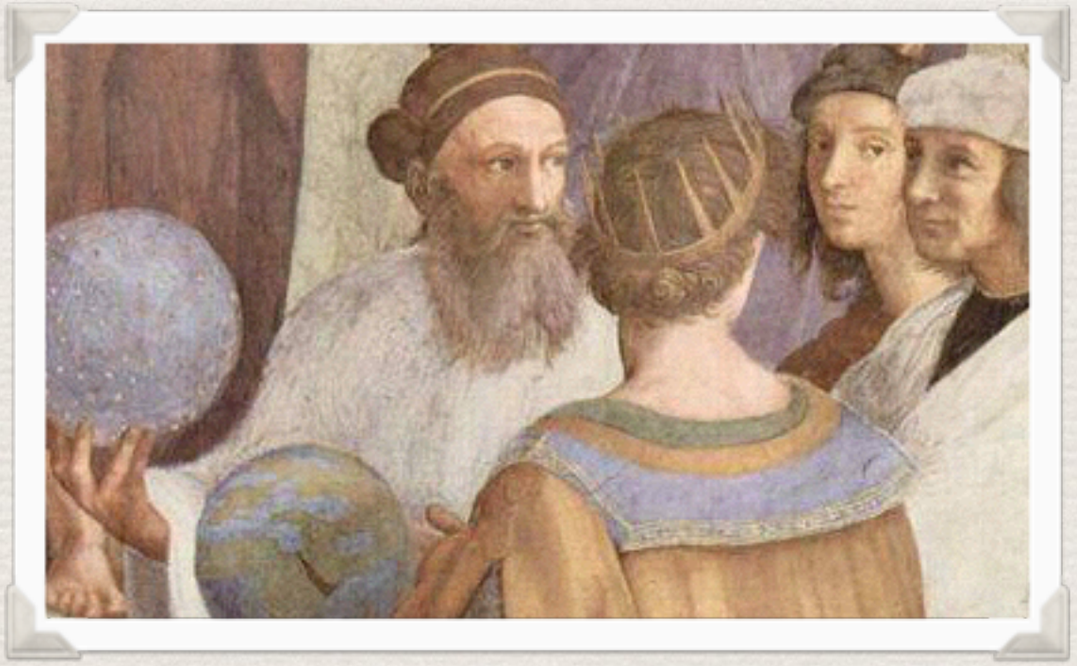
I.e., after 19 tropical years, the Sun and Moon return to the same position on the ecliptic.

Solar Anomaly

	Meton (432 BCE)	Callipus (330 BCE)	Hipparchus (120 BCE)
Summer Solstice to Autum Equinox	90 days	92 days	92 ½ days
Autumn Equinox to Winter Solstice	90 days	89 days	88 ⅛ days
Winter Solstice to Spring Equinox	92 days	90 days	90 ⅛ days
Spring Equinox to Summer Solstice	93 days	94 days	94 ½ days

Qualitative Meets Quantitative

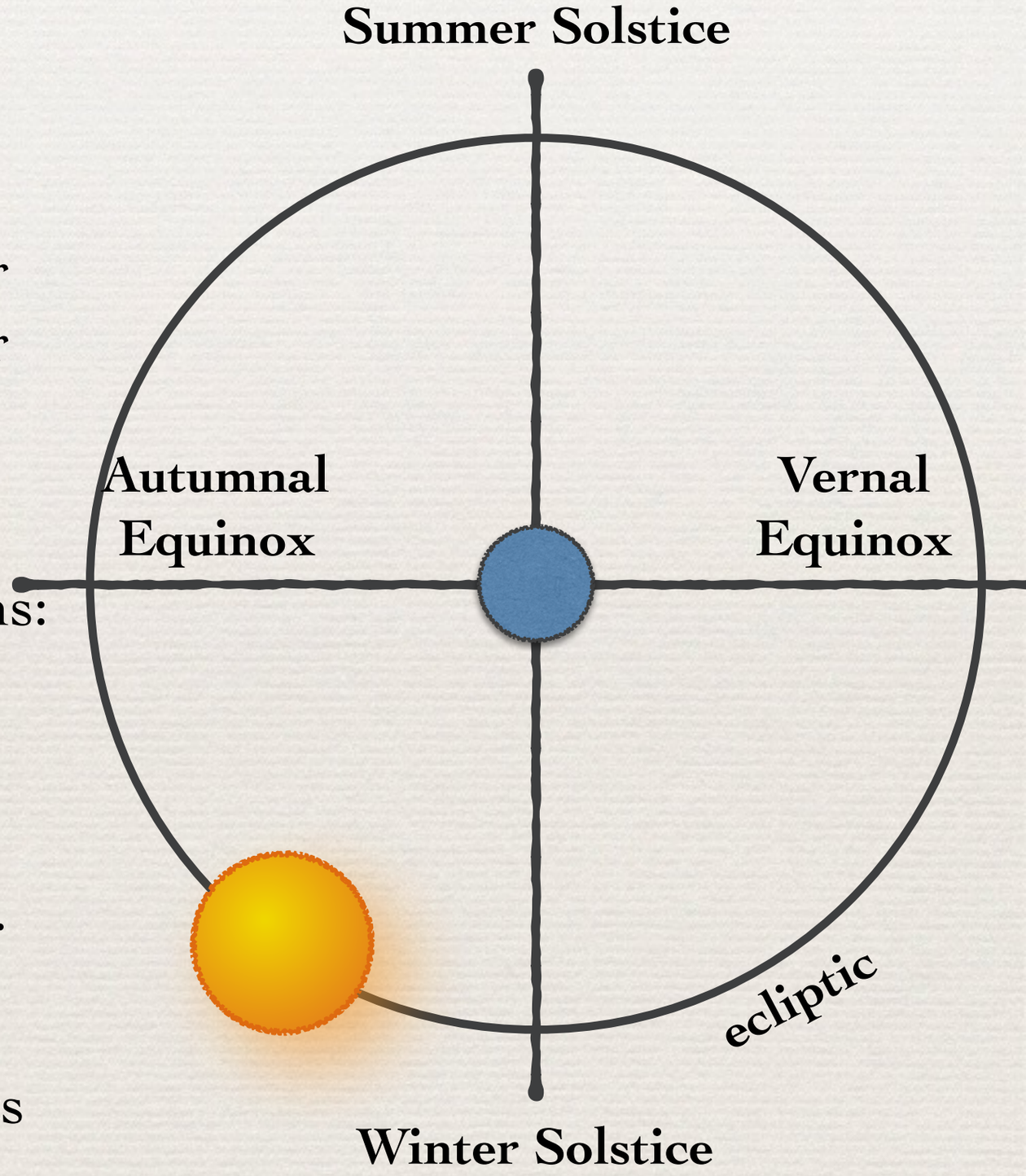
- Hipparchus continues the Greek tradition of developing explanatory geometric models, but was strongly influenced by Babylonian astronomy.
    - Geometric models can and should be adapted or modified to agree with past data and become predictively accurate.



Hipparchus (190 - 120),
as depicted in *The School of Athens* (1511)

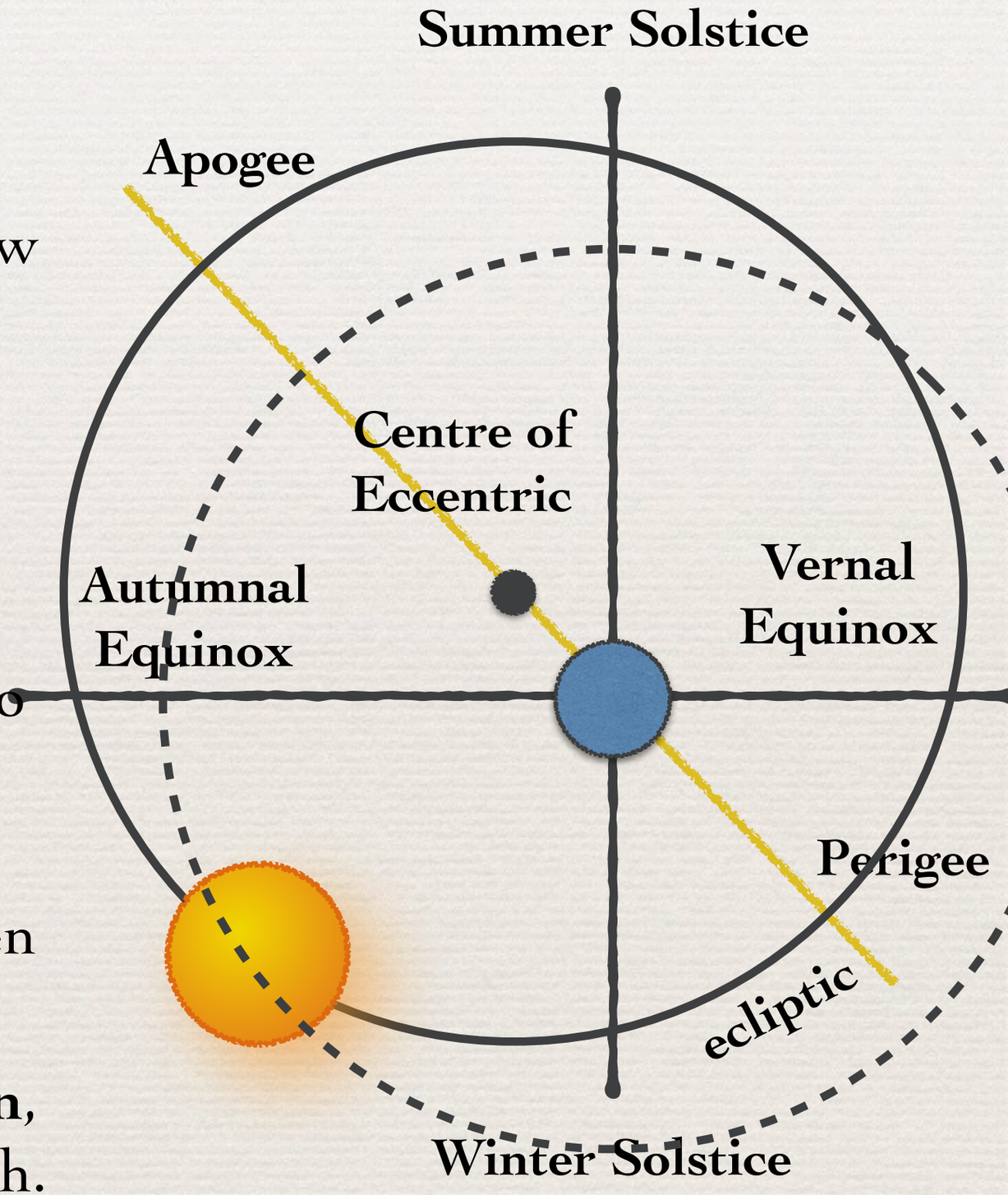
Solar Theory

- Consider the Sun:
  - The naive nested-sphere model for seasonal changes, but not the solar anomaly.
- To account for this, Hipparchus had to give up at least one of three assumptions:
  1. The Sun's orbit is a circle
  2. Centred on the Earth
  3. The Sun travels at a uniform speed.
- Giving up any violates Aristotelian physics; but giving up 1 or 3 also results in great computational complications.

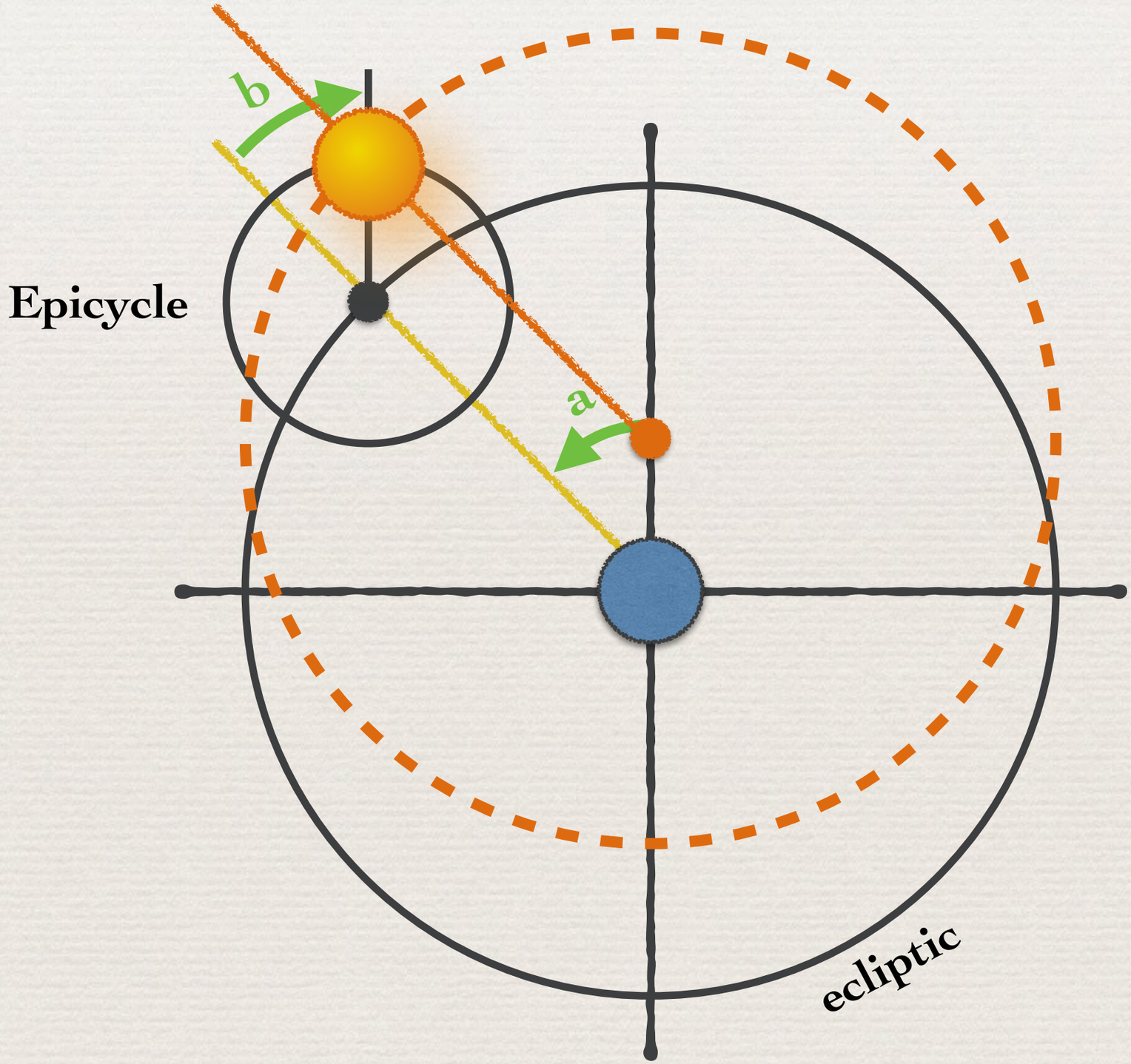


Solar Theory

- Hipparchus' solar theory has four parameters, which once determined allow for accurate predictions:
  - The length of the tropical year
  - The longitude of the apogee (which Hipparchus took to be fixed)
  - The ratio of the distance from Earth to the Centre of the Eccentric to the radius of the orbit
  - The longitude of the Sun at some given moment.
- A useful concept is that of the **Mean Sun**, which moves uniformly around the Earth.



An Alternative Theory…



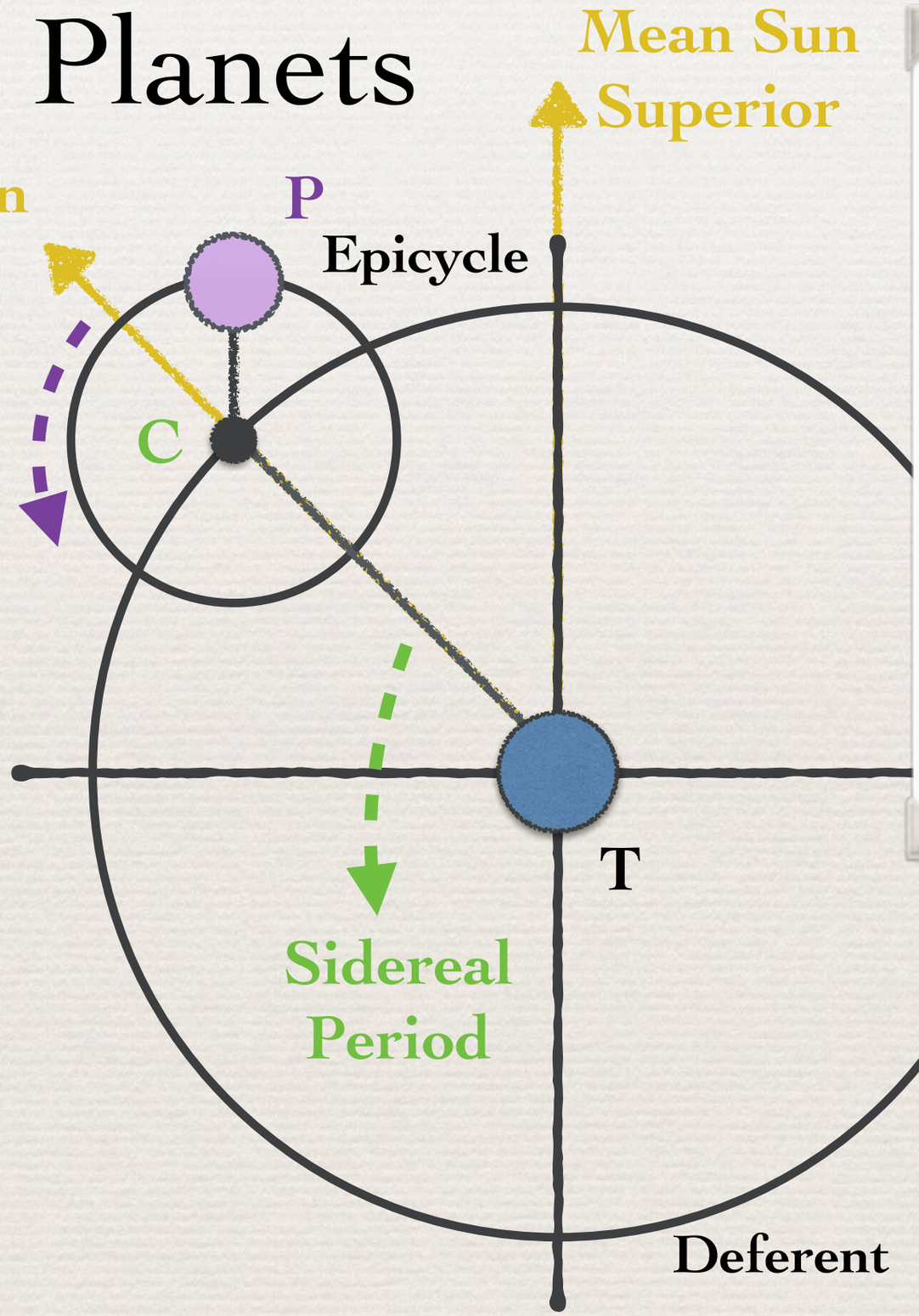
An Alternative Theory…

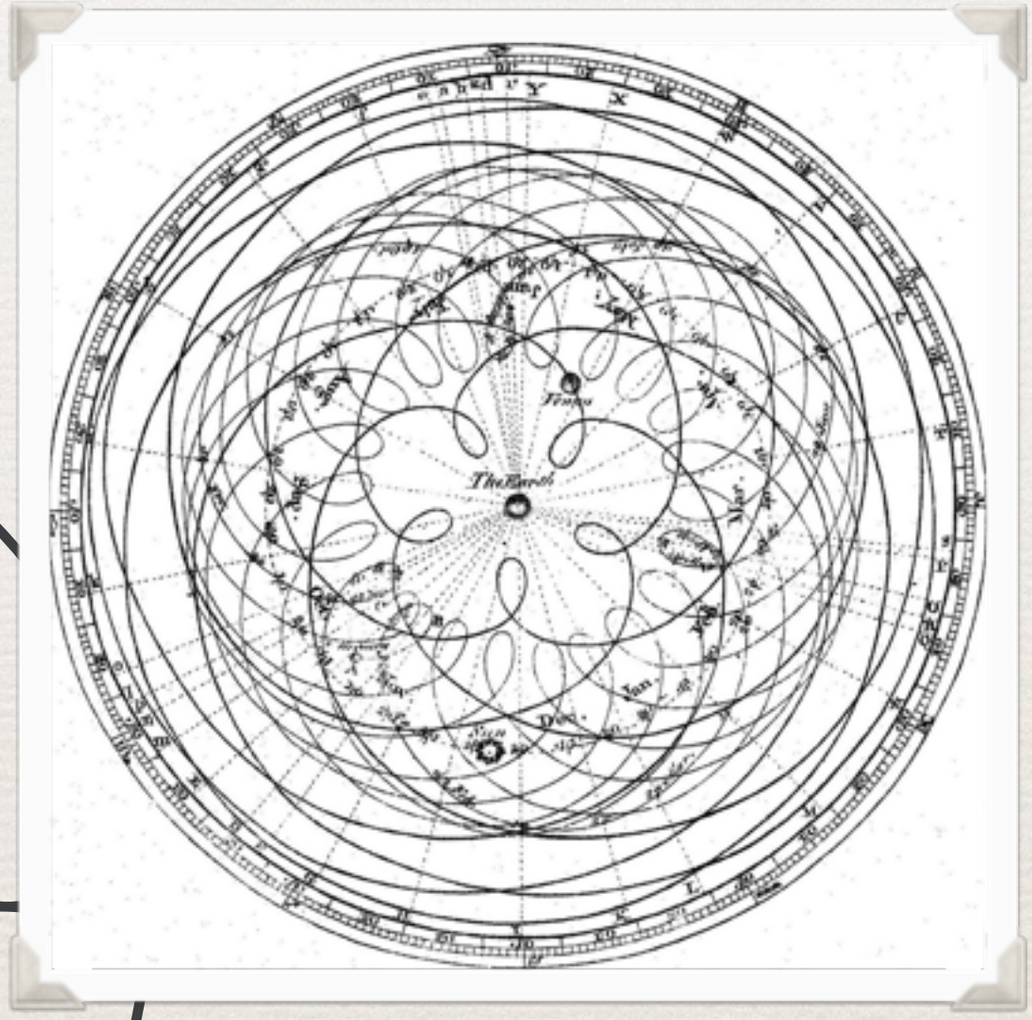
- It was remarkable to the Greeks that two theories so physically different end up being mathematically equivalent.
  - **Hipparchus** preferred the **Epicycle model**—It is probable that celestial bodies were placed uniformly with respect to the centre of the cosmos.
  - **Ptolemy** preferred the **Eccentric model**—It is "simpler", in that it involves one motion instead of two.
- As with Aristotle before, we see these scholars grappling with the notion that they should be trying to determine the physical structure of the universe.
  - And constrained/guided by the methodological precept of reducibility to uniform circular motion.

- Lived and worked in Alexandria, writing a substantial compendium of astronomy.
  - The ***Almagest*** — thirteen books, nearly 500 pages in modern translation.
    - Title is an Arabic corruption of ***Megiste Syntaxis*** (Greater Compendium)
    - By far the most influential and important book in Medieval astronomy.
- Theory of the Sun is the same as Hipparchus, computing the length of the year from two vernal equinoxes and two summer solstices — the oldest reported by Meton in 432 BCE.
    - Checking the accuracy of Ptolemy's observations, we find him more than a day out regarding his own observations — suggest that he did not actually make observations, but simply computed the dates based upon Hipparchus' values.
    - Also explains why he got the same value as Hipparchus for longitude of the Sun's apogee — $65\frac{1}{2}°$ — by Ptolemy's time it was 71°.

Claudius Ptolemy
(100 - 170 CE)

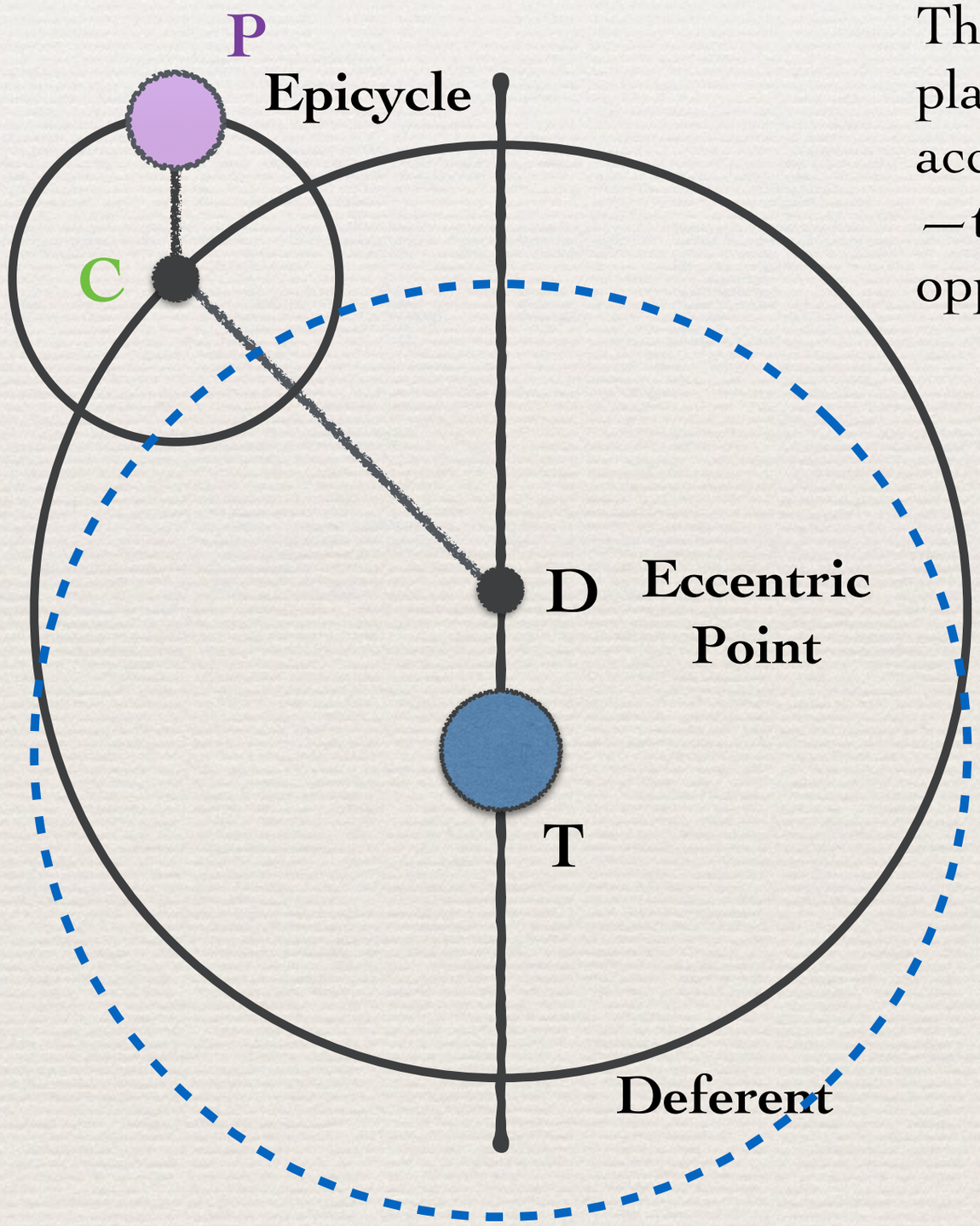
The Planets





Note that the meaning of "Synodic Period" has changed: Rather than the interval between two retrogradations, it's now the time to go once around an epicycle.

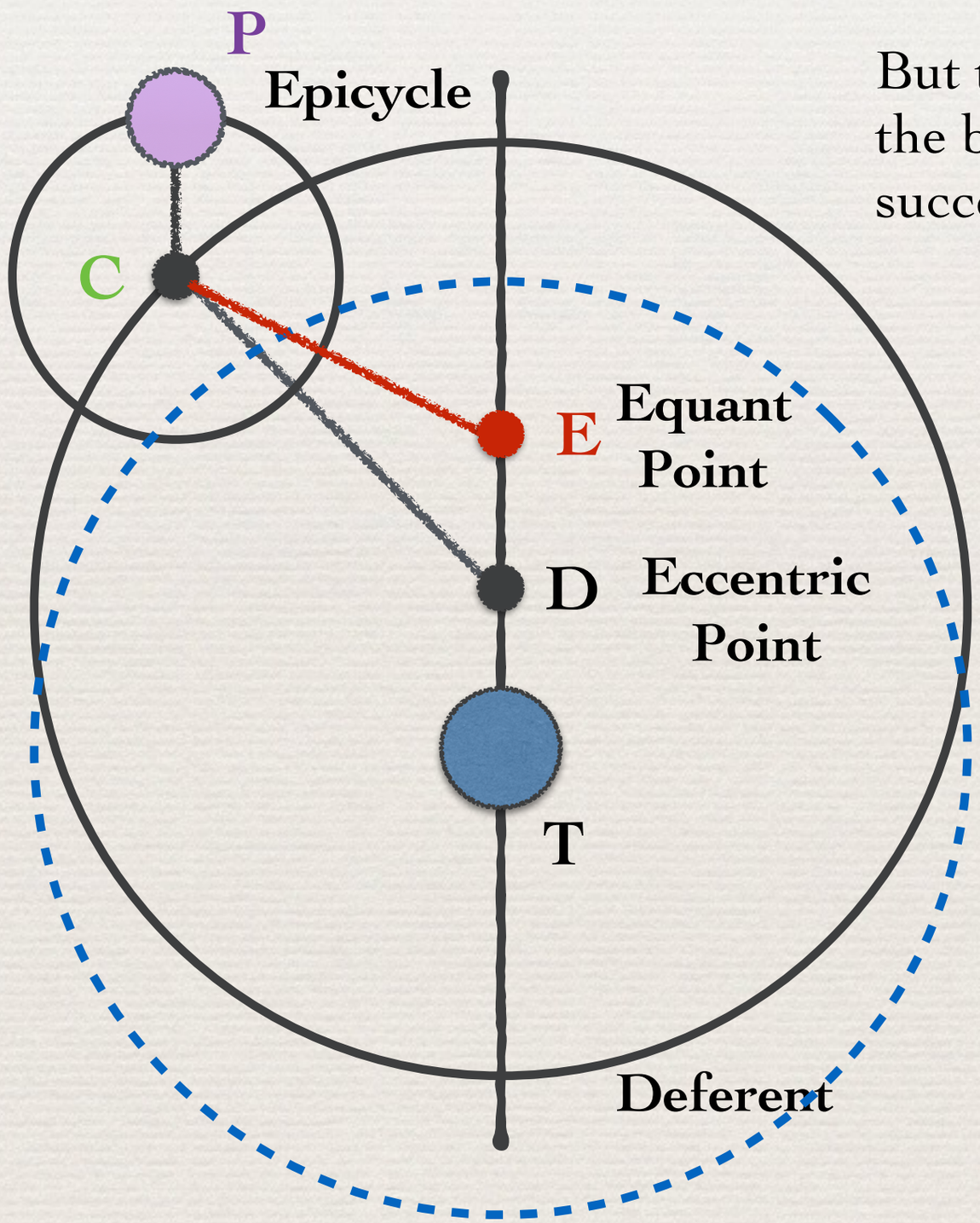
The Planets… Details



The length of the arc covered by a planet in it's retrogression varies according to its position on the ecliptic —the position where it is longest is opposed to where it is shortest.

The eccentric accounts for this by making the arc appear larger at perigee and smaller at apogee.

The Planets… Details

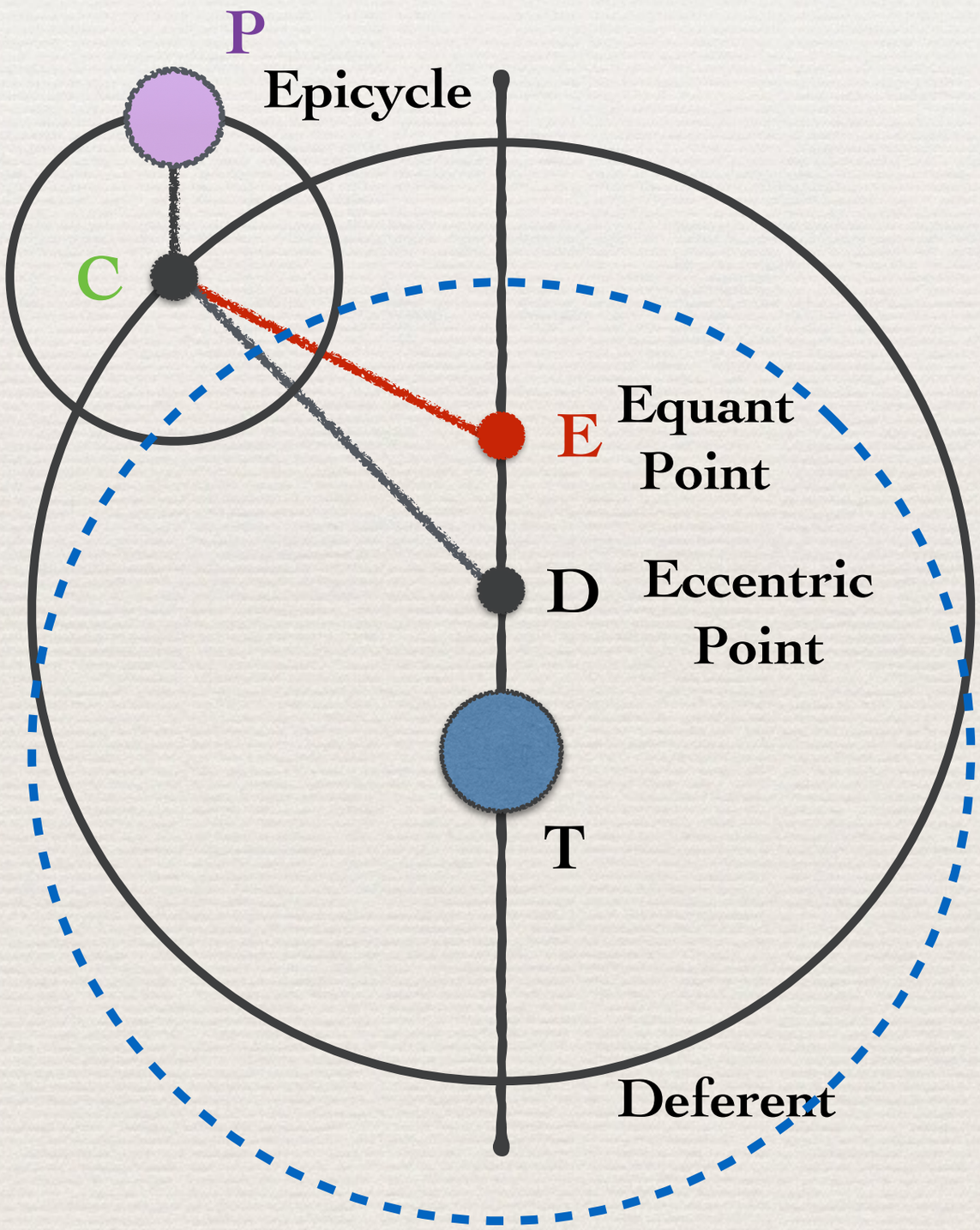


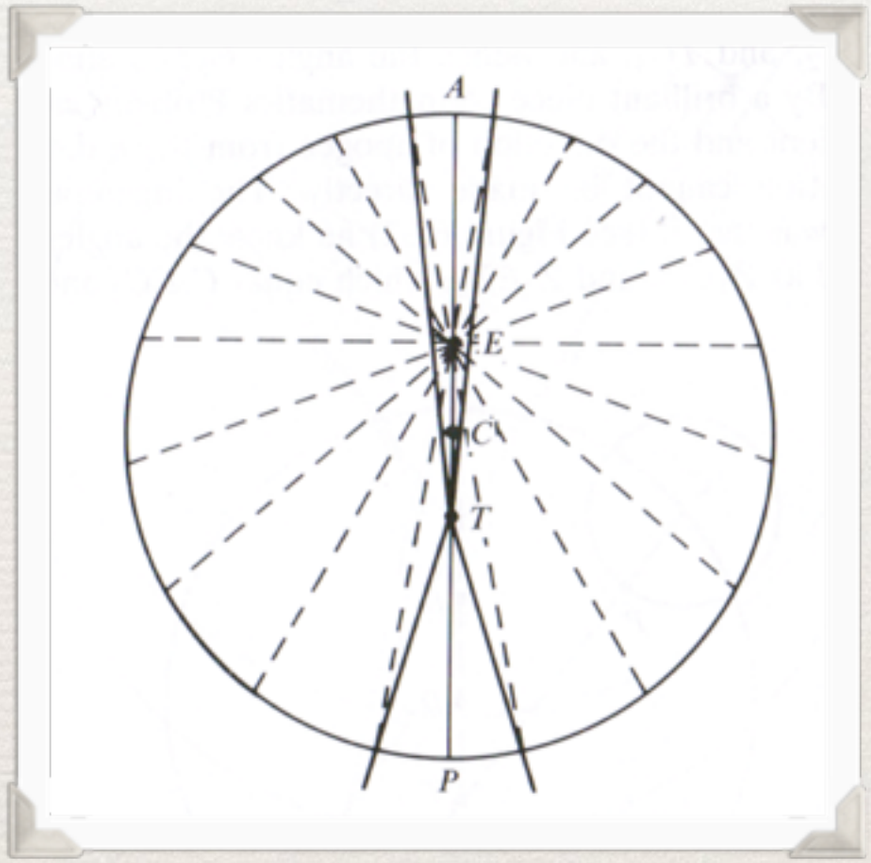
But the interval on the ecliptic between the beginnings (or endings) of two successive retrograde arcs also varies.

This indicates that the mean speed of the planet changes—the zodiacal anomaly is greatest at perigee and slowest at apogee.

The correct length for TD to account for variation in retrograde arcs, accounts for only half the variation in orbital speed.

The Planets… Details





Hugh Thurston (1994), *Early Astronomy*. Springer-Verlag

How Accurate?

	Average Error Longitude	Maximum Error Longitude
Mercury	3°	7°50′
Venus	1°	At least 4°30′
Mars	25′	55′
Jupiter	10′	30′
Saturn	25′	50′

- Note that Ptolemy's theory as described here is in two dimensions—a different theory in the final book of the *Almagest* deals with deviations from the ecliptic.

Problems…

- The single biggest problem with this system was that it failed to hold account to the methodological precept of **Uniform Circular Motion**.
  - And that it disagreed with Aristotelian physics.
- There seems no **reason** as to why the planets will vary their speed throughout the course of their orbits.
- Ptolemy argues that we keep a kind of Uniform Motion around the equant.

Claudius Ptolemy
(100 - 170 CE)

Consider now how great these difficulties are. ***If what Aristotle has stated with regard to natural science is true,*** *there are no epicycles or eccentric circles and everything revolves round the centre of the Earth. But in that case how can the various motions of the stars come about? […]*

Furthermore, how can one conceive the retrogradation of a star, together with its other motions, without assuming the existence of an epicycle? On the other hand, how can one imagine a rolling motion in the heavens or a motion around a centre that is not immobile? ***This is the true perplexity.***

Moses Maimonides
(1135 - 1204)

However […] this does not affect the astronomer. For his purpose is not to tell us in which way the spheres truly are, ***but to posit an astronomical system in which it would be possible for the motions to be circular and uniform and to correspond to what is apprehended through sight****, regardless of whether or not things are thus in fact. […]*

[R]egarding all that is in the heavens, ***man grasps nothing but a small measure of what is mathematical;*** *[…] The heavens are the heavens of the Lord, but the Earth hath He given to the sons of man. I mean thereby that the deity alone fully knows the true reality, the nature, the substance, the form, the motions, and the causes of the heavens.*

- Copernicus' prime motivations were to remove the equant point, and to find a definite centre for the cosmos—a physical model more **plausible**.

- It would not be until **Johannes Kepler** (1609), that astronomy would free itself of the methodological precept of Uniform Circular Motion and Aristotelian physics.

Nicolaus Copernicus
(1473 - 1543 CE)

Thanks!